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БИФУРКАЦИЯ ЭНЕРГИИ СВЯЗИ И ХАОС В АТОМНЫХ ЯДРАХ

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В работе рассмотрена модель хаотического поведения нуклонов в атомных ядрах, построенная на основе модели ядерных взаимодействий и статистики Ферми-Дирака

Ключевые слова: НЕЙТРОН, ПРОТОН, ФЕРМИ-ДИРАКА СТАТИСТИКА, ХАОС, ЭНЕРГИЯ СВЯЗИ, ЯДРО

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BINDING ENERGY BIFURCATION AND CHAOS IN ATOMIC NUCLEI

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The model of chaotic behavior of nucleons in nuclei, based on the model of nuclear interactions and the Fermi-Dirac statistics is discussed

Keywords: BINDING ENERGY, CHAOS, FERMI-DIRAC STATISTICS, PROTON, NEUTRON, NUCLEI

It is known, that the binding energy of nucleons in atomic nuclei depends on a regular motion of protons and neutrons in the nuclear shells, and on the chaotic behaviour of nucleons, which correlates with uncertainty in the measurement of the mass of the nuclides [1-3]. The concept of quantum chaos [4-5] is the basic model of chaotic behaviour of the nucleons.

We consider the model of the bifurcation of the binding energy in atomic nuclei, based on the generalized dynamics of the Verhulst-Ricker-Planck equation [6]. To derive the equations of the model the results of the theory of strong interactions of nucleons in nuclei [7-8] used. According to this theory there is a relationship between the size of the nucleus, binding energy and the interaction parameter, which can be written as follows:

$$r_n E = \sqrt{Sb_{nl}^{A}} = b(A)A \tag{1}$$

Here, $A = N + Z$ - the number of nucleons (protons + neutrons), as the units used the speed of light, Planck constant and electron mass. The binding energy is determined by the number of nucleons with a total mass of proton and electron, thus $E = A(m_p / m_e + 1) - M_A / m_e$.

Since equation (1) must be shared with the standard expression of the size of the nucleus, $r(A) = r_0 A^{1/3}$, reflecting the weak compressibility of nuclear matter, we can define the left-hand side of equation (1) using experimental data

[9]. As a result, we find the product of the binding energy and nuclei radius depending on the number of nucleons - Fig. 1. For consistency with the data [9], we put

$$b(A) = 0.05325 \ \ln A \, .$$

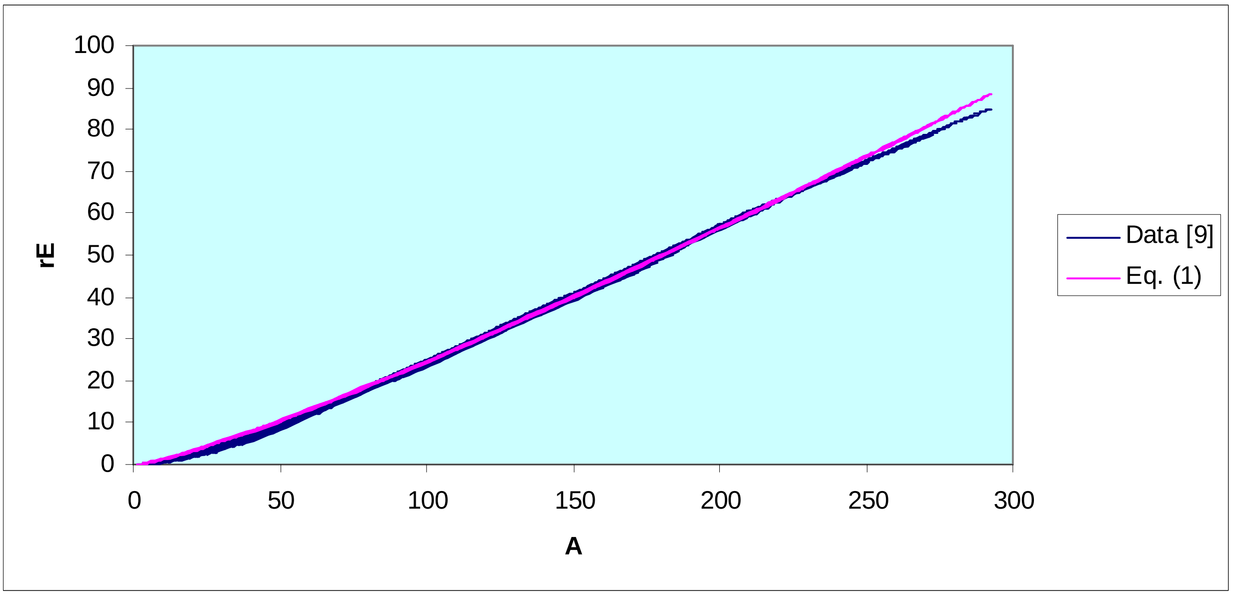

Fig. 1: The product of the binding energy and nuclei radius depending on the number of nucleons according to [9].

Using this correlation, we can represent equation (1) as

$$E_A = bA / r_A \tag{2}$$

Now we can construct a discrete model of the energy levels in nuclei as follows:

$$E_{A+1}E_A^2 = \frac{b(A+1)(A+1)(b(A)A)^2}{r_{A+1}r_A^2} =$$

$$\frac{A}{4pr_A^3/3}\,\frac{4pr_A}{3Ar_{A+1}}\,b(A+1)(A+1)(b(A)A)^2$$

(3)

On the other hand, the density of nucleons can be related to the binding energy due to Fermi-Dirac statistics, we have

$$n_A = \frac{A}{4pr_A^3/3} = \frac{g_Z Z/A}{e^{(E_Z - m_Z)/q} + 1} + \frac{g_N N/A}{e^{(E_N - m_N)/q} + 1} \qquad (4)$$

Here g_i, E_i, m_i, q are the weight factors, energy and chemical potential of protons and neutrons, and the statistical temperature of the nucleon, respectively. Model (3) - (4) was investigated in a wide range of parameters. Let us consider the results obtained in the simplified model under the condition of equality of chemical potentials and energy of the two types of nucleons

$$m_N = m_Z = m_A = q \ln a, \quad E_Z = E_N = -E_A / A.$$

In this case, the model can be written as

$$\begin{aligned} x_{A+1} x_A^2 &= \frac{K_0 (1 + 1/A)^{2/3} b^2(A) b(A+1)}{e^{-x_A} + a} \\ x_A &= \frac{E_A}{Aq}, \quad K_0 = \frac{4p}{3Aq^3} a g_A \\ g_A &= g_N + g_Z; \quad b(A) = 0.05325 \ln A \end{aligned} \qquad (5)$$

To close the model (5), it is necessary to formulate the law of temperature and the weight factor change with the number of nucleons. We use a simple hypothesis, which follows from the theory of the Fermi gas of elementary particles [10] that these parameters are proportional to the cube of the boundary momentum, which in turn is determined by the size of the system:

$$q = k_1 p_0^3, \; g_A = k_2 p_0^3, \; p_0^2 = k_3 / r_A \qquad (6)$$

Hence, we find that the temperature and the weight factor decreases with increasing number of nucleons as follows

$$q = T_0 A^{-1/2}, \quad g_A = g_0 A^{-1/2} \qquad (7)$$

Under conditions (6)-(7), the parameter K_0 on the right side of equation (5) does not depend on the number of nucleons. Let us consider the behaviour of the chemical potential depending on the number of nucleons. Above we assume that the chemical potentials of protons and nucleons are equal and, moreover, their relation to temperature is a constant, which coincides with the logarithm of the fine structure constant. To test this hypothesis, let consider functions

$$f(A) = -m_A / q \ \ln 137$$
$$f_0(T_0) = \sum_{A \geq 12} f(A) / \sum_{A \geq 12} A(N, Z) \qquad (8)$$

Using data [9] and equations (5) - (7), we can calculate functions (8) – see Fig. 2-3. Data [9] plotted in Fig. 2-3 show that the chemical potential of nucleons reaches the theoretical value $m_A = q \ln a$ for the number of nucleons over 12 and for $T_0 > 20$ MeV . Note that the chemical potential of the bound nucleon system is negative, whereas the chemical potential of free fermions is positive and limited by the Fermi energy at zero temperature - see [10-11].

There is a critical point at $T_0 \approx 20$ MeV as it shown in Figure 3. We suggest that real nuclides have a temperature over critical temperature. Therefore a constant in eq. (7) determined and a linear dependence of chemical potential and temperature established.

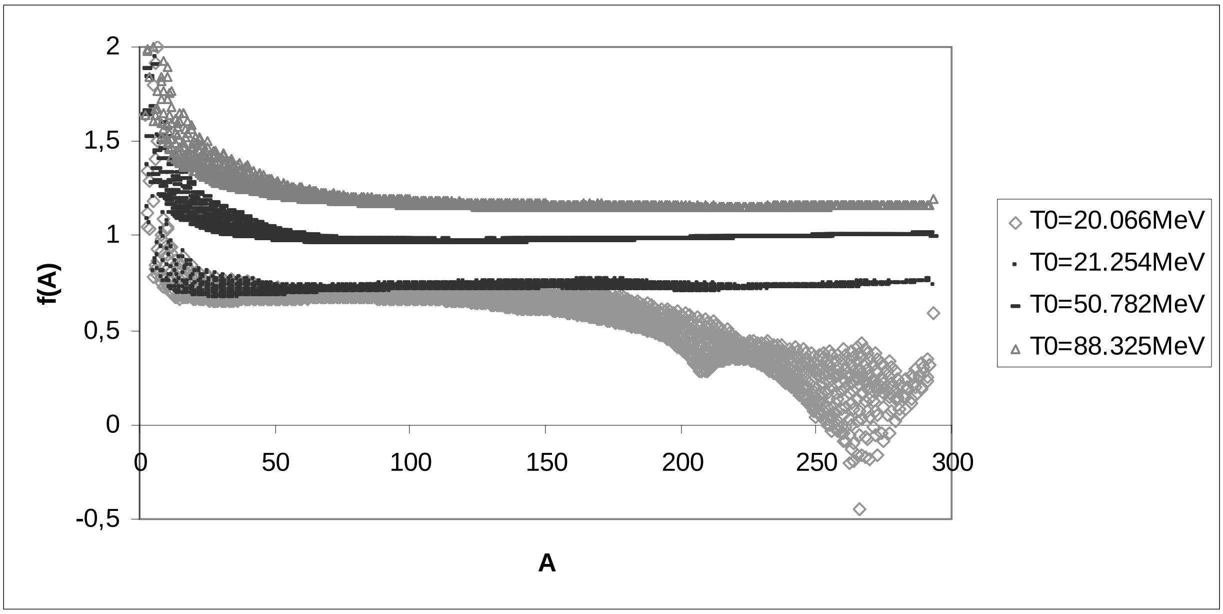

Fig. 2: Chemical potential over temperature as a function of the number of nucleons, calculated on equations (5) - (7), and data [9]. $f(A) = -m_A / q \ \ln 137$.

For light nuclei, the chemical potential, as well as other parameters of the model (5)-(7) deviates from the theoretical dependence (6). Nevertheless, we use the model (5), starting with the deuterium nucleus contains two nucleons. We set the starting point at $x_2 = 0.2$. As a result, we find that the structure of energy levels, which is implemented in a system of nucleons with higher temperature $T_0 = 52.858\ MeV$ - Figure 4. In this case, the first bifurcation point for the binding energy of light nuclei corresponds to the carbon isotope ^{12}C, and the second bifurcation point - nickel isotope ^{58}Ni.

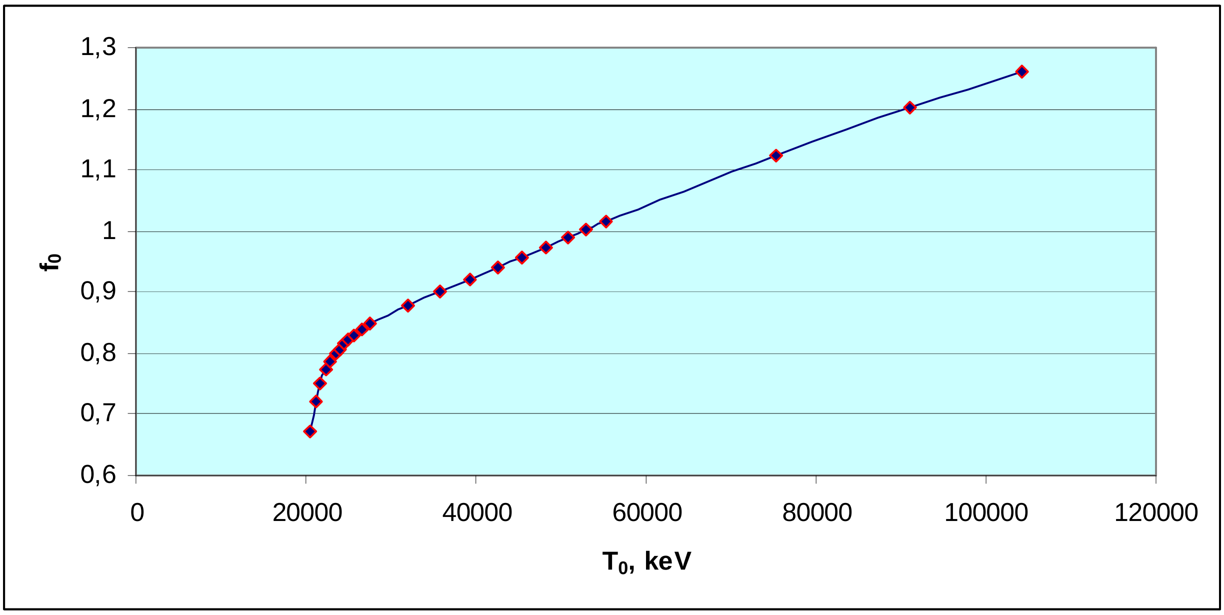

Figure 3: The chemical potential parameter as a function of T_0.

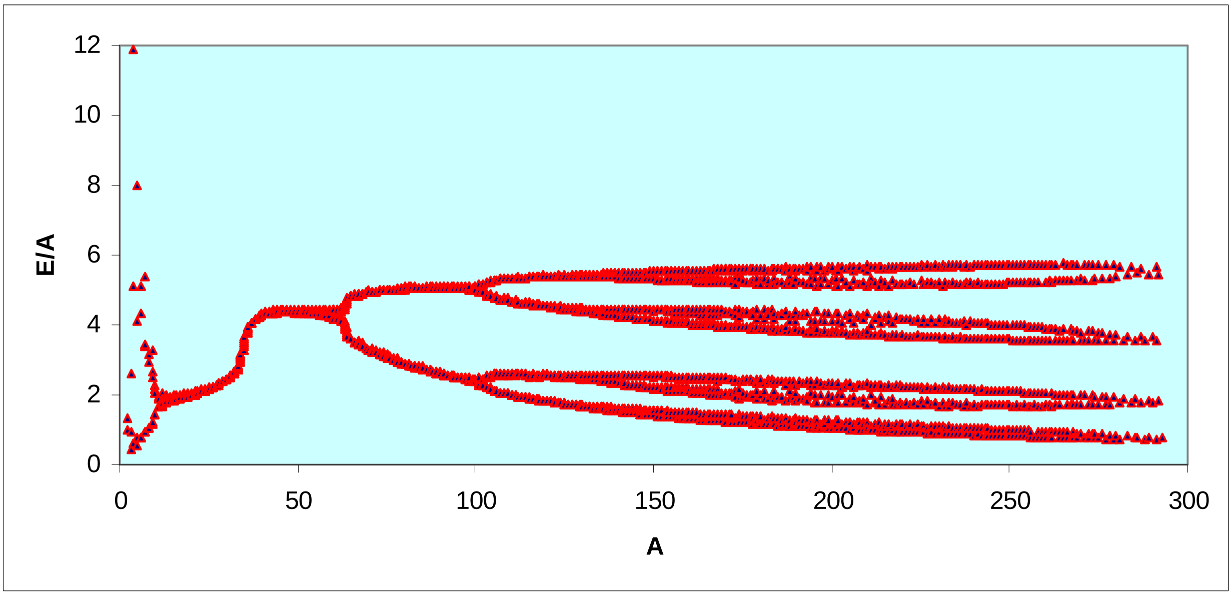

Figure 4: Binding energy per nucleon as a function of mass number calculated on eq. (5)-(7) at $a = 1/137$; $K_0 = 0.0371$.

With the number of nucleon increasing the energy levels are split series at 2, 4, 8, 16 sublevels, as shown in Figure 4. The specific structure "four rats", first observed in [6], is formed by increasing parameter K - Figure 5. It was also shown in [6] that there is the transition to chaotic behaviour in a model (5) in the region $a \le 1/137$.

It was established that the transition to chaotic behaviour in a model (5) is also observed in violation of the equality of chemical potentials of the two kinds of nucleons – Figure 6. If the chemical potentials of protons and neutrons are strong differ, than the structure shown in Figure 7 forming, which superficially similar to the experimental dependence - Fig. 8.

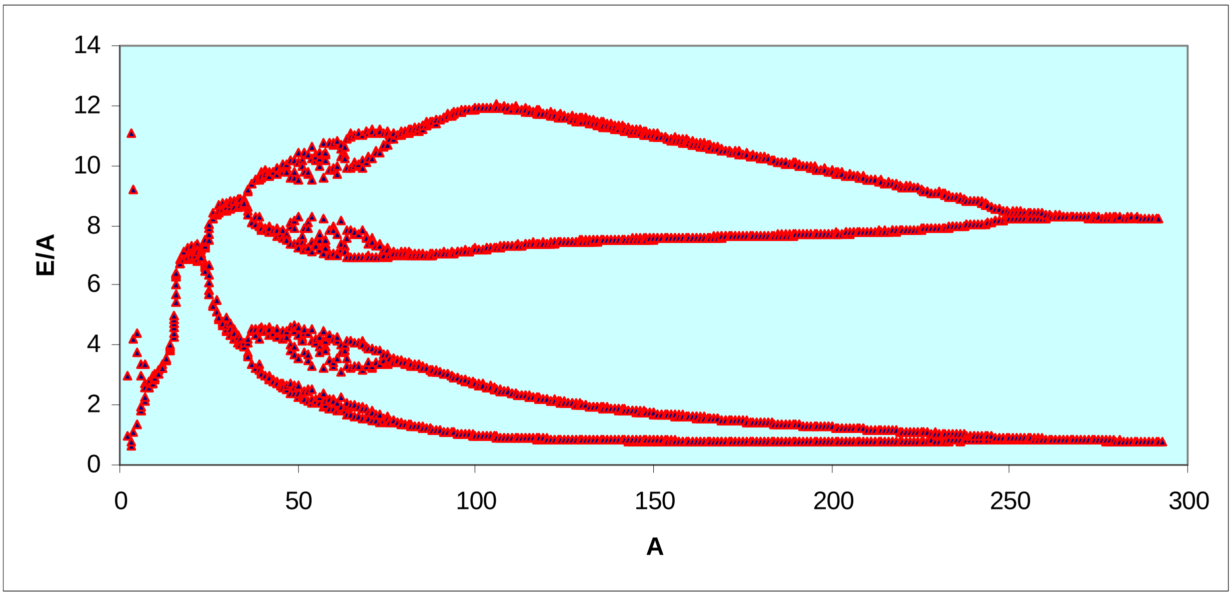

Figure 5: Binding energy per nucleon as a function of mass number calculated on eq. (5)-(7) at $a = 1/137$; $K_0 = 0.0839$.

Let us give an interpretation of the results. Model (3) - (7) is a thermodynamic one. It shows how the binding energy changing if one nucleon in the nuclei added, taking into account changes in density according to the Fermi-Dirac distribution at finite temperature. It is well known that the binding energy of nucleons in the nucleus depends on the number of neutrons and protons. Standard semi-empirical formula describing the binding energy is given by [11]

$$E_b = a_1 A - a_2 A^{2/3} - a_3 Z(Z-1) A^{-1/3} - a_4 (N-Z)^2 A^{-1} + a_5 A^{-3/4}$$

(9)

$$a_1 = 14;\ a_2 = 13;\ a_3 = 0.585;\ a_4 = 19.3;\ a_5 = 33 d(A, N, Z)$$

Here are shown current values of the coefficients derived from data [9]. All coefficients are given in MeV. In this expression, the function $d(A, N, Z)$ is defined as:

$d = 1$ for even Z, N;

$d = -1$ for odd Z, N;

$d = 0$ in all other cases.

The first and fourth term on the right side of expression (9) depend on the kinetic energy of nucleons, which is calculated on the basis of statistics (4) at zero temperature [11]. However, the data in Fig. 2 and eq. (6) - (8) show that temperature not zero and the chemical potential can be varied with temperature by other way than theory of the Fermi gas of free particles predicts, like it explained in [10-11] and other university books. In particular, the chemical potential in a system of nucleons in nuclei is negative, as well as the binding energy.

There is a minimal constant $T_0 \approx 21 MeV$ for which is still running a linear relationship of temperature and chemical potential in the area $A > 12$ - Fig. 2. Consider the solution of equation (5) in the case $T_0 = 21 MeV$ - Fig. 8. There are two bifurcation points $A = 25; 148$, between which the calculated curve attached to data [9]. One branch of the solution diverges in the region $A > 300$, while the other vanishes. We can assume that in real nuclei $T_0 \approx 21 MeV$, that agrees with the value $a_4 = 19.3 MeV$ in the semi-empirical equation (9). Further studies will show whether it is possible to predict binding energy on model (3)-(4) with accuracy exceeding the semi-empirical equation (9).

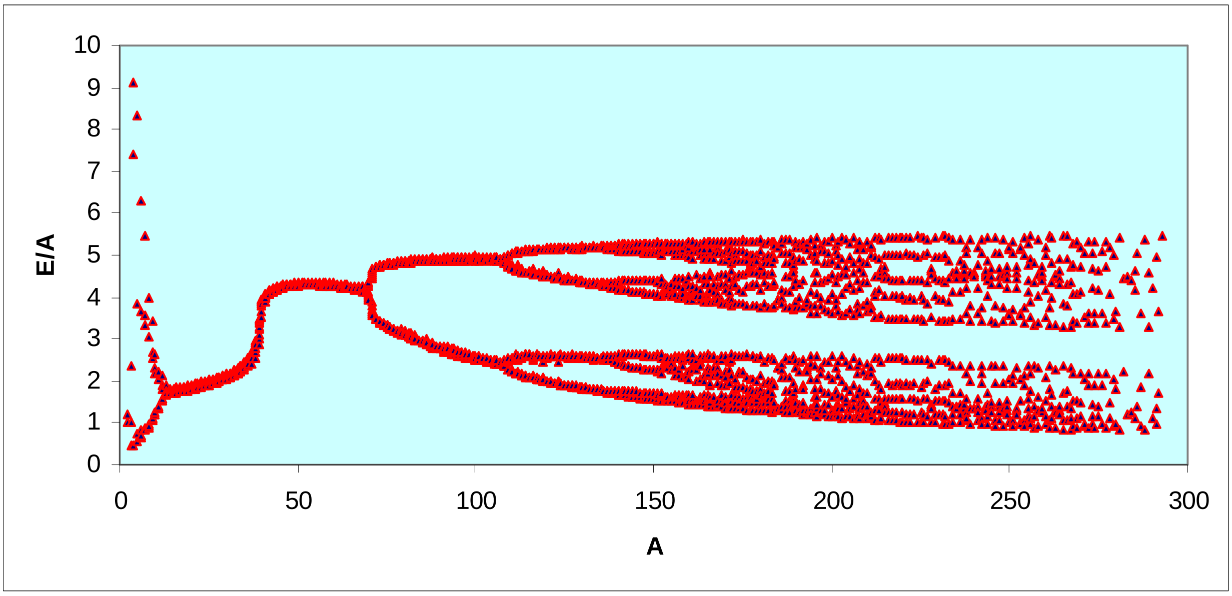

Figure 6: Binding energy per nucleon as a function of mass number calculated on eq. (5)-(7) at $K_0 = 0.063$; $m_p / q = -\ln(137)$; $m_n / q = -\ln(171)$.

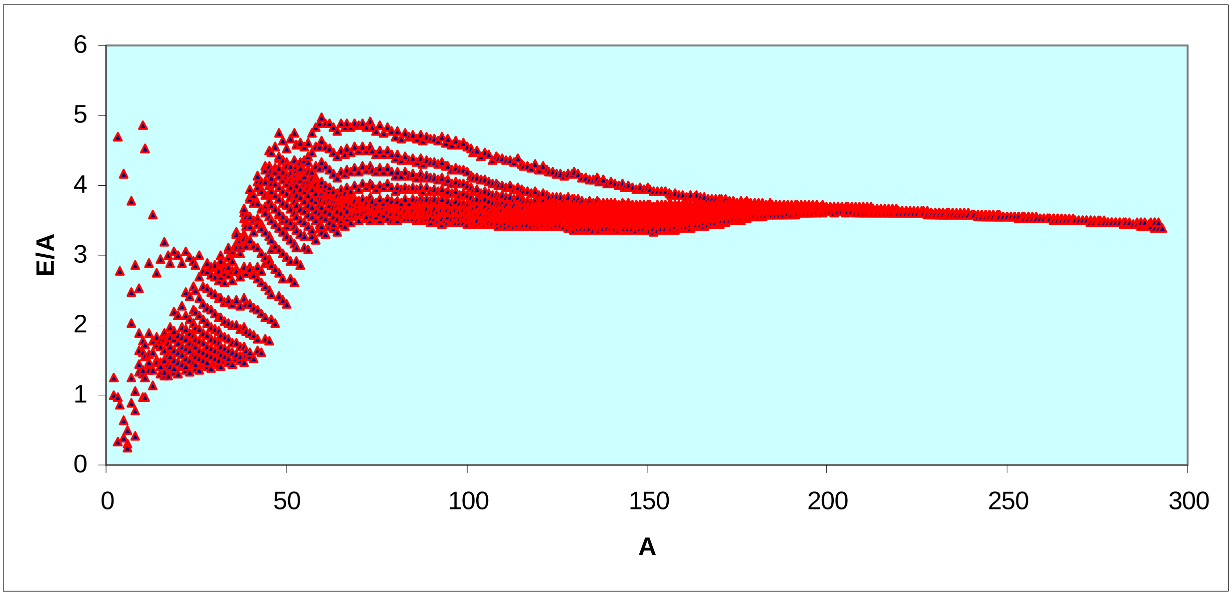

Figure 7: Binding energy per nucleon as a function of mass number calculated on eq. (5)-(7) at $K_0 = 0.07$; $m_p / q = -\ln 137$; $m_n / q = -2\ln 137$.

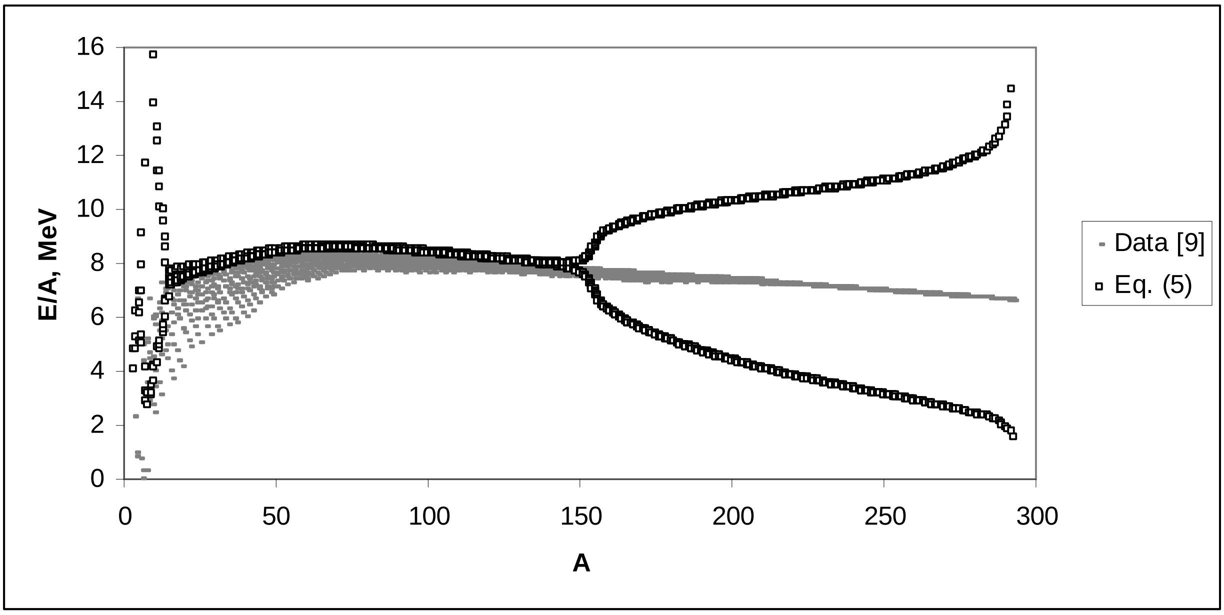

Figure 8: Binding energy per nucleon as a function of mass number calculated on eq. (5)-(7) at $T_0 = 21\,MeV$; $K_0 = 0.03623$.

Equation (5) is a nuclei statistical model describing the dynamics associated with changes in the number of fermions at nonzero temperature. It also can be used for a given number of nucleons in standard form [6] as follows

$$
\begin{aligned}
& x_{i+1} x_i^2 = \frac{K}{e^{-x_i} + a} \\
& x_i = \frac{E_i}{Aq}, \quad K = \frac{4p}{3Aq^3} a g_A (1 + 1/A)^{2/3} b^2(A) b(A+1)
\end{aligned}
\tag{10}
$$

Eq. (10) is iterated from $x_0 = 1$ for set of K up to asymptotic stable state $x = x(K)$. Bifurcation diagram of eq. (10) plotted in double logarithmic coordinate is shown in Figure 9. Apparently it correlates with data in Figure 5 and it has own name “four rats” [6]. Main result concerning structure “four rats” is that there is transition to chaos in a region $a \le 1/137$ - Figure 10. It looks like the fine structure constant $a = e^2 / \mathbf{h} c = 1/137.0359990$ could be calculated from model (5) as a transition point between regular and chaotic behaviour of nucleons in a nuclei.

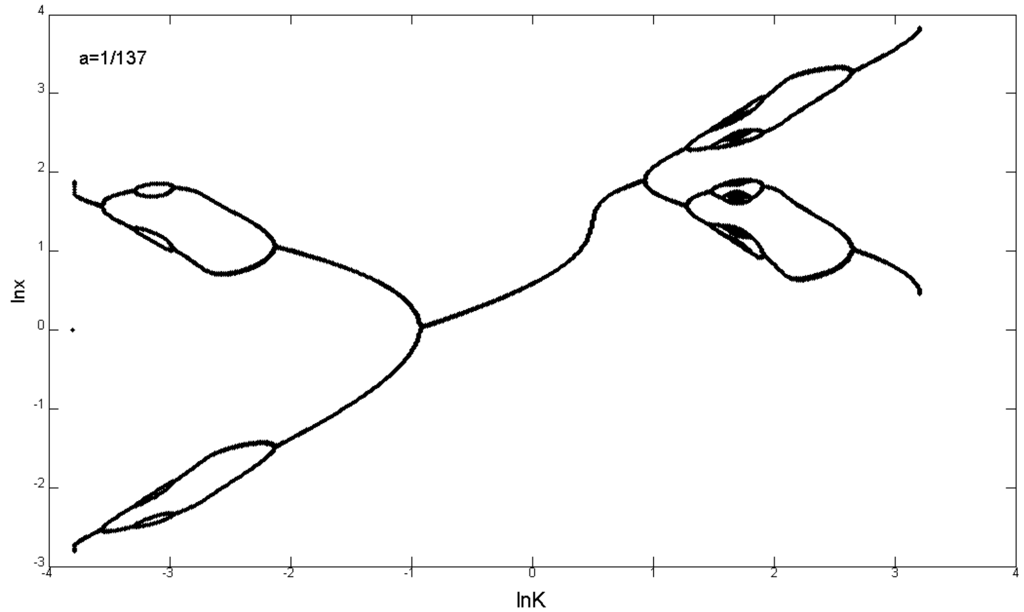

Figure 9: Bifurcation diagram “four rats” calculated on model (10).

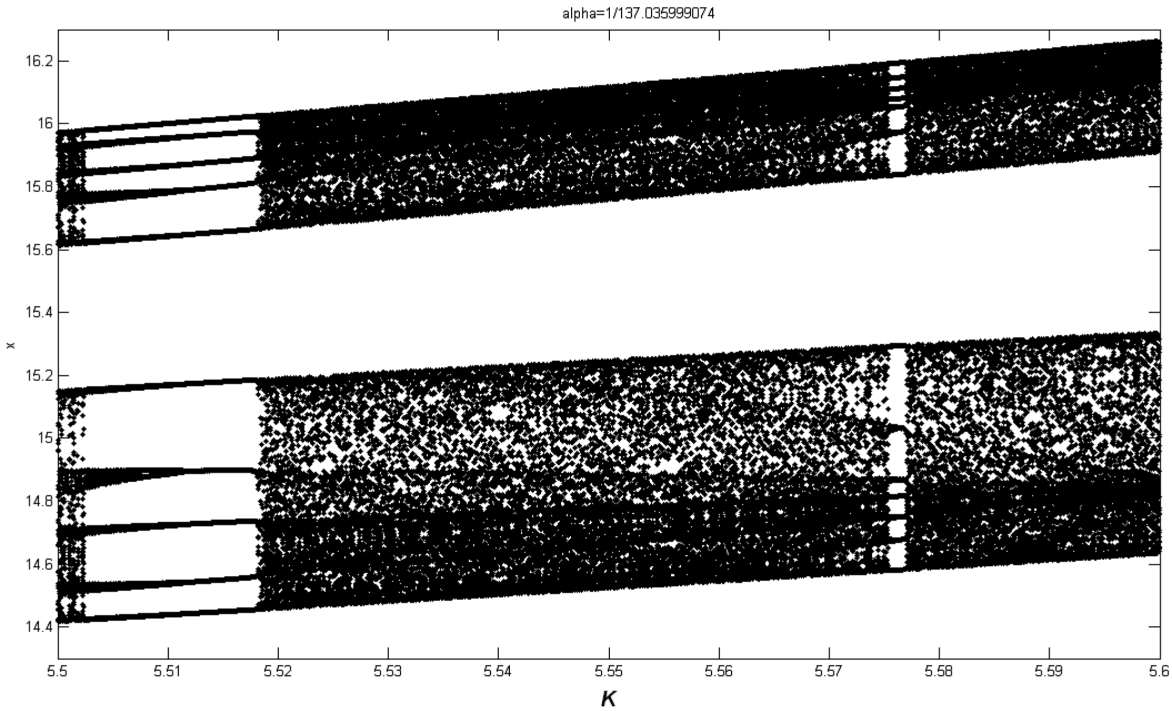

Figure 10: Fragment of bifurcation diagram “four rats” demonstrating a chaos in “rat ears”.

The obtained results on the chaotic behaviour of the nucleons in the bound system indicate the complexity of describing the state of the nuclei, since the splitting of energy levels can occur not only due to the dynamic conditions imposed by the presence of nuclear interaction [7-8], and nucleons dynamics [12-13], but also due to statistical reasons related to the influence of temperature in accordance with statistics of fermions [1].

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