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ТЕОРИЯ ТУРБУЛЕНТНОСТИ И МОДЕЛИРОВАНИЕ ТУРБУЛЕНТНОГО ПЕРЕНОСА В АТМОСФЕРЕ
ЧАСТИ 1, 2

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В работе представлена полностью замкнутая модель турбулентного пограничного слоя, полученная из уравнения Навье-Стокса. Фундаментальные константы пристенной турбулентности, включая постоянную Кармана, определены из теории. Эта модель была развита для ускоренного и неизотермического пограничного слоя над шероховатой поверхностью. Исследованы численные решения системы уравнений турбулентного преноса примесей в приземном слое атмосферы для больших масштабов

Ключевые слова: АТМОСФЕРНАЯ ТУРБУЛЕНТНОСТЬ, ТУРБУЛЕНТНЫЙ ПЕРЕНОС, УСКОРЕННЫЕ ТЕЧЕНИЯ, ПОГРАНИЧНЫЙ СЛОЙ, ШЕРОХОВАТАЯ ПОВЕРХНОСТЬ, ПРИЗЕМНЫЙ СЛОЙ АТМОСФЕРЫ, ТУРБУЛЕНТНЫЙ ПЕРЕНОС АЭРОЗОЛЕЙ

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THEORY OF TURBULENCE AND SIMULATION OF TURBULENT TRANSPORT IN THE ATMOSPHERE
PARTS 1, 2

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The completely closed model of wall turbulence was derived directly from the Navier-Stokes equation. The fundamental constants of wall turbulence including the Karman constant have been calculated within a theory. This model has been developed also for the accelerated and non-isothermal turbulent boundary layer flows over rough surface. Numerical solutions of equations system of turbulent transport of admixtures in a surface layer of the atmosphere for a large scale have been studied

Keywords: ACCELERATED FLOW, AEROSOL TURBULENT TRANSPORT, ATMOSPHERIC STRATIFIED FLOW, ATMOSPHERIC TURBULENCE, ATMOSPHERIC SURFASE LAYER, BOUNDARY LAYER, ROUGH SURFACE, TURBULENT TRANSPORT

CONTENTS

1. Introduction

Lately an interest for the problems of estimation of atmosphere air quality in the cities has increased. Firstly it should be connected with the policy of environment protection which is carried out by developed countries. In 1992, II UN Conference on Environment protection and development adopted the Declaration which enunciated principles of stable development. In particular a strategy has been worked out on limitation and reduction of carbon dioxide emission which can effected catastrophically on the world climate. In 1993 in Russia a national plan on environment protection and stable development was worked out. The main principles of this plan became the basic in the policy of management in the regions [2]. In these declarations and plans atmospheric air is considered a resource, qualitative consumption of which should be guaranteed not just for the present generation of human beings but for future generations as well.

The problem of air pollution is tightly connected with the development of industry, transport and energetic. Continuous process of coal, natural gas and organic fuel burning aimed at obtaining electrical energy and heat, wide spread of automobile transport, waste of chemical plants and metallurgical works – all this leads to accumulation of different chemical compounds in the atmosphere, which affect the atmosphere composition in a planetary scale. Waste of nitric oxide, sulphur oxide and carbon oxide badly affect different components of biosphere. Health of the urban population in big industrial cities is aggravating because of air pollution. It has been found that air pollution results in building and monuments destruction. Among natural factors of air pollution there are volcanic activity and wind erosion, the latter is partially connected with agricultural development: extension of land under crops and soil destruction because of intensive exploitation.

Practically all industrial countries exercise environment control, in the sense, that waste of harmful substances should be limited, including the atmosphere. For the effective management a special control of air quality is needed, which includes a certain waste volume limitation, taking into consideration the current composition of atmosphere. In its turn air composition is determined by continuous and periodical measurements in the local points, as well as at mobile stations i.e. by monitoring. Even a well-organised network of the observation stations cannot secure true information of the air quality in the whole physical volume in a given region. It is necessary to have a suitable model in order to apply data of observations to the areas where monitoring stations are not available. Thus, the problem arises of mathematical modelling of the quality of free air, tightly connected with a solution of a problem of diffusion of admixtures in the atmosphere under a given emission [3].

Thanks to acknowledging by the community and government, the importance of solution of the problem of ensuring permanent monitoring of air pollution, such powerful research trends as EUROTRAC-program which includes over 250 research groups in 24 European countries [4] – have been supported. It should be noted that the main aim of EUROTRAC – is co-ordination of programs of scientific researches of transboundary transport and chemical transformation of admixtures in the troposphere over Europe.

Mathematical modelling of the air quality is getting more and more effective instrument in the analysis of atmosphere condition, due to the rapid development of electronic computers and decrease in their cost, and also perfection of mathematical models of transport of gaseous fluid- and solid dispersed components of pollution. Systems of free air quality modelling have been created in big cities such as Paris [5], Lisbon [6], Budapest [7], on a planetary scale [8], and over huge regions as Western Europe or northern latitudes of Eurasia [9-10], as well as in small towns such as Oxford or Cambridge [11] and even in the central street of London [12].

The mane peculiarity of the above mentioned models [5-12] and others is that modelling of admixtures transport is carried out on the base of systems of diffusion equations with coefficients depended on parameters of atmospheric turbulence. It should be noted that modelling of air flows in the boundary layer of atmosphere is a complicated problem, solution of which depends on the theoretical ideas about turbulence. Thus, a simplified parameterization of planetary boundary layer [see 3, 8, 10], which allows completely eliminate calculations of complicated atmospheric flows, using data of meteorological parameters – is a popular method in description of global transport of admixture in the atmosphere.

But this approach cannot be realized in solution the problem of transport of admixture in the lower layers of atmosphere, which is very important for the urban air quality control.

Indeed, transboundary transport of admixtures influences mainly a formation of background value of air pollution level in the regions, while to determine the local level of pollution it is necessary, first, to take into consideration local centres of emission. But among all sources of pollution, automobile transport in most regions is an outright winner in the way of intensive pollution of the atmosphere. For example, traffic contribution into air pollution in Sochi (Russia) makes up over 80% [2, 13-14] that can be compared with Athens (75% from total mass of wastes NOx) and London (63% NOx) [15].

So, a problem of transport of admixtures into the areas having one or more highways is one of the priorities. Typical space scale in such task should be several hundreds meters from axis of the road and dozens meters in height. Thus the process of turbulent diffusion of traffic wastes is localized in the surface layer of atmosphere and greatly depends on parameters of turbulence as well as on the condition of stratification. Besides heterogeneity of ground surface, including artificial roughness in a form of buildings, trees, etc [16] is a very important factor in this problem.

Turbulent transport of moment, heat and mass in a surface layer greatly depends on heat convection [17]. Due to a great scale of atmospheric flows, convection in the surface layer is observed in a kind of turbulent motions of complex structure [18-20]. Nevertheless a turbulent flow in a thermal stratified surface layer is considered as a spatial homogeneous steady flow with well determined average parameters, which depend only on a coordinate across a boundary layer. Above mentioned approach to the description of turbulent flows in a stratified flow has been developed by Monin & Obukhov [21, 22]. Monin and Obukhov’s similarity theory has been further developed and has got experimental confirmation by numerous researches [23-27].

Modern models of turbulent transport of admixtures in the lower part of atmosphere are based, mainly, on different modifications of $k-e$ model [28-29], which in its turn represents expansion of mean according to Reynolds equations of Navier-Stokes for the purpose of their closing. Some closures for turbulent stratified flow and with due regard for the planet rotation have been presented in [30-35]. A review of the main methods of atmospheric boundary layer modelling is given by authors [36-38].

As it is known, the main idea in the description of mean turbulent motions of a fluid consists in representations of a vector of flow velocity as a sum of vectors of average, by time, velocity of flow and velocity of pulsate motion: $\mathbf{u} = \langle \mathbf{u} \rangle + \mathbf{u}'$, where by definition

$$\langle \mathbf{u} \rangle = \frac{1}{\Delta t} \int_{t}^{t+\Delta t} \mathbf{u}(\mathbf{x}, t') dt' \tag{1.1}$$

The averaging time interval in (1.1) is suggested quite enough to exclude the chaotic pulsate movement of a fluid. Using the averaging procedure (1.1) in the

Navier-Stokes model one can derive the momentum equation for turbulent incompressible flow as follows

$$\frac{\partial U_i}{\partial t}+\frac{\partial U_i U_j}{\partial x_j}+\frac{1}{r}\frac{\partial P}{\partial x_i}=n\nabla^2 U_i+\frac{1}{r}\frac{\partial t'_{ij}}{\partial x_j} \quad (1.2)$$

where $U_i=\langle u_i\rangle$, $P=\langle p\rangle$ is the mean pressure, $t'_{ij}=-r\langle u'_i u'_j\rangle$ is the Reynolds stress tensor.

This approach was developed over 100 years ago by O. Reynolds [39], who presented equations of turbulent flows (1.2), widely used nowadays in engineer application as, for example, models [28-29].

It should be noted that according to Reynolds vectors $\langle \mathbf{u}\rangle,\mathbf{u}'$ are not supposed to be of any solutions of the Navier-Stokes equations. So, resultant equations (1.2) in common case are unclosed and thus additional physical ideas are needed to close the model. In the simplest cases model closures for incompressible flows in a boundary layer is carried out the base of the Boussinesq's conception of eddy viscosity [40]. Thus for a turbulent flow in OX direction we have

$$t'_{xz}=-r\langle u'_x u'_z\rangle=n_t\frac{\partial U_x}{\partial z} \quad (1.3)$$

where z is the coordinate normal to the wall

Prandtl [41-42] offered a handy formula of turbulent viscosity, using a parameter of the mixing length, by analogy with the length of molecules run, as follows

$$n_t = rl^2\left|\frac{\partial U_x}{\partial z}\right| \quad (1.4)$$

where l is the mixing length.

The initial Prandtl’s model (1.4) is notable for its extraordinary simplicity, because the parameter of a mixing length is estimated out of independent hypotheses based on the similarity theory. Prandtl’s ideas turned out to be fruitful and constructive, so most of the models of turbulent flows in the boundary layer, in any case, are based on the hypotheses of eddy viscosity and a mixing length.

Kolmogorov [43] suggested more general expression of eddy viscosity, considering it dependent on an average kinetic energy of turbulence, which can be estimated on the base of hypothetical equations, in the form

$$t'_{xz}=-r\langle u'_x u'_z\rangle= rn_T\frac{\partial U_x}{\partial z};\ n_T=C_m l_k k^{1/2} \quad (1.5)$$

where C_m is the constant, l_k is the turbulent length scale, $k=\langle u'_k u'_k\rangle/2$ is the turbulent kinetic energy.

Launder ***et al*** [28] suggested another form of expression (1.5) introduced the turbulent dissipation rate $e_T = k^{3/2}/l_k$, thus with this parameter the eddy viscosity (1.5) can be rewritten as

$$n_T = C_m k^2 / e_T \tag{1.6}$$

The various modifications of the $k-e$ model are based mainly on the eddy viscosity model (1.6). The standard $k-e$ model is included the continuity equation for the mean velocity, the momentum equation (1.2), the turbulent transport equations for 'substances' k, e_T, and a generalised form of the first equation (1.5) for Reynolds stress tensor:

$$-\left\langle u_i' u_j' \right\rangle = n_T \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \frac{2}{3} k d_{ij} \tag{1.7}$$

The main problem, peculiar to the approach mentioned above, is difficulty to control various physical effects which are the key in the process of turbulent transport. As far as, according to Reynolds point of view, functions $\langle \mathbf{u} \rangle, \mathbf{u}'$ are not the solutions of initial equations, their physical sense is not obvious enough to connect them with the forces functioning in the system. So, model of turbulence [28-35] as well as some others are notable for numerous of different constants and parameters introduced for co-ordination of estimated values with experimental data.

So, in a case of atmospheric stratified flows, following Rodi [29], $k-e$ model can be written as

$$\frac{\partial U_i}{\partial x_i} = 0 \tag{1.8}$$

$$\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} + \frac{1}{r} \frac{\partial P}{\partial x_i} = n \nabla^2 U_i + \frac{1}{r} \frac{\partial t_{ij}'}{\partial x_j} + g_i \frac{r - r_0}{r_0}$$

$$\frac{\partial T}{\partial t} + U_j \frac{\partial T}{\partial x_j} + \frac{\partial}{\partial x_i} \left(\left\langle u_i' T' \right\rangle - \frac{n}{\Pr} \frac{\partial T}{\partial x_i} \right) = 0$$

$$-\left\langle u_i' u_j' \right\rangle = n_T \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \frac{2}{3} k d_{ij}, \quad -\left\langle u_i' T' \right\rangle = \frac{n_T}{\Pr_t} \frac{\partial T}{\partial x_i}$$

where r_0 is the reference density, g_i is the gravitational acceleration, T, T' are the mean temperature and temperature fluctuations respectively, $\Pr_t$ is the turbulent Prandtl number.

The closures for model (1.8) can be derived by the common method explained by Rodi [29], as follows

$$\frac{\partial k}{\partial t} + U_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_i}\left(\frac{n_T}{s_k} \frac{\partial k}{\partial x_i} \right) + \Pi + \Gamma - e_T \tag{1.9}$$

$$\frac{\partial e_T}{\partial t} + U_j \frac{\partial e_T}{\partial x_j} = \frac{\partial}{\partial x_i}\left(\frac{n_T}{s_e} \frac{\partial e_T}{\partial x_i} \right) + C_{1e}\frac{e_T}{k}(\Pi + C_{3e}\Gamma) - C_{2e}\frac{e_T^2}{k}$$

where Π, Γ are stress and buoyancy production of the turbulent kinetic energy, respectively. The last terms can be expressed as

$$\Pi = n_T \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \frac{\partial U_i}{\partial x_j}; \quad \Gamma = b g_i \frac{n_T}{\Pr_t} \frac{\partial T}{\partial x_i} \tag{1.10}$$

here $b = -r^{-1}\left(¶ r / ¶ T \right)_p$ is the coefficient of expansion, $b = 1/T$ for the perfect gases.

Model (1.8-1.10) depends on the six parameters C_{1e}, C_{2e}, C_{3e}, $\Pr_t, s_e, s_k$. Two of them, C_{3e} and the turbulent Prandtl number, $\Pr_t$, are not constants for arbitrary stratification, but its dependent on the stability parameter.

As it has been shown by Apsley & Castro [44] the $k-e$ model requires modification in the atmospheric boundary layer. For instance, in a case of a neutral stratification, the standard $k-e$ model (1.8) - (1.10) yields a very deep boundary layer and large friction velocity. They supposed the new form of the $k-e$ model extended for the case of a stable stratification.

Thus, in general case the problem of closing the models of turbulent transport has not got a comprehensive solution, because the parameters of atmospheric turbulence depend on roughness of ground surface, local heat flux, pressure-gradient and velocity of the planet rotation (or Coriolis forces). So, a turbulent flow in the boundary layer of the atmosphere is a three-dimensional and not stationary and can be described in particular cases only, for example, in surface layer, in which the conditions of homogeneity and quasi stationary for the mean turbulent fluxes of the heat and momentum are carried out (these hypotheses are the bases of the Monin-Obukhov's similarity theory [21-22]).

The aim of this work is development of the theory of turbulent diffusion of admixtures in the lower part of atmosphere, which can be applied to describe the flows with arbitrary stratification. In the work process it became vivid that hypotheses about influence of buoyancy forces and roughness of ground surface on turbulent boundary layer should have been revised. It became possible mainly due to simplicity of description of external forces within the theory of turbulence [45-48].

In this paper the basic principles of the developing theory of turbulence are given and the idea of mechanisms of influence of static and dynamic roughness on turbulent flows parameters in the boundary layer is substantiated. The averaging method of turbulent flow parameters in the boundary layer has been suggested, application of which allows distinguishing among all solutions of Navier-Stokes equations the functions, similar, according to the characteristics, to those which are to be observed in experiments. Formulas are given of hydrodynamic equations transformation to curvilinear nonstationary system of coordinates, connected with chosen surface, which is modelled of the dynamic roughness. The formal transformation of hydrodynamic equations in the surface layer is described. On the base of the equations of dynamics of viscous, heat-conducting fluid and an equation of diffusion, by application of transformations of the type mentioned, the system of dynamic equations for random amplitudes of flow parameters has been obtained: admixture velocity, pressure, temperature and concentration. For realistic modelling of atmospheric flows, the buoyancy forces are taking into consideration, produced by thermal expansion of the air, for the description of which Boussinesq approximation is adopted. The analysis of similarity of the system of equations in the case of steady, non-isothermal flow in longitudinal pressure gradient has been accomplished.

2 Theory of turbulence

2.1 Formal principles of theory

Modern models of turbulent flows are mainly based on the Navie-Stokes equations averaged according to Reynolds as it was mentioned in Introduction. It is clear, that any model, based on presentation of the velocity field as a sum of mean velocity and velocity of pulsate motion: $\mathbf{u} = \langle \mathbf{u} \rangle + \mathbf{u}'$, in which vectors $\langle \mathbf{u} \rangle, \mathbf{u}'$ are not the solution of initial equations, should not be closed. And this is the main problem of turbulent flows modelling – mean equations are not adequate to initial Navie-Stokes model, and therefore are not complete. Further locking of equations obtained, is practically based on experimental data interpretations, so models of turbulent flows are notable for excess adjusting parameters which are introduced to register an effect of real forces, functioning in the system. In spite of notable success in this sphere, Cantwell [52] passed a remark that with the exclusion of some results obtained from reasons of dimensionality the simplest problems for turbulent flow with the simplest out of possible boundary conditions, cannot be solved up to now. Therefore the methods of direct numerical simulation (DNS) of turbulent flows have been developed for the last decade [53], but their application to the atmospheric boundary layer is still a problem due to a lack of resources of modern computers [54].

It should be noted that if vector $\langle \mathbf{u} \rangle$ is somehow a solution of initial equations, then the model which has been obtained by Reynolds' method, will be closed without any other additional hypothesis. In this case the main target is a search for a suitable solution, $\tilde{\mathbf{u}}$, the characteristics of which should be conformed to the experimentally estimated profile of the mean velocity. Algorithm of finding such type of solutions for the flows in a turbulent boundary later was suggested in the papers [45-48]. It consists in transformation of Navier-Stokes initial equations so, that they would contain eddy viscosity, as in the Prandtl theory of mixing length. Let's consider this algorithm in detail.

Note that the field of velocity in a turbulent flow is not a regular function, but it is not a random function as well, because it doesn't depend on random parameters. The main idea is to introduce the random parameters into the hydrodynamic equations, the flow velocity evidently depends. This is possible in a particular case of flows in a boundary layer, for which a surface layer transformation is performed, i.e. presentation of a flow velocity vector, $\mathbf{u} = (u, \mathrm{v}, w)$, as $\mathbf{u} = \mathbf{u}(x, y, z/h(x, y.t), t)$, where z is the normal to the wall variable, $h = h(x, y, t)$ - is the dynamic roughness surface adjoining to the wall but not coinciding with it. Physical interpretation and the model of the dynamic roughness surface will be given below.

The dynamic roughness surface in a turbulent flow is advised to characterise by the set of the random continuous parameters h, h_t, h_x, h_y , which have a meaning of the height, inclination and velocity of transport of surface elements, with a well-known function of distribution $f_s = f_s(h, h_x, h_y, h_t)$.

We suppose that $h = z / h = const$ and consider a representative region of flow by a volume $dV = L_x L_y dz$, where L_x, L_y are typical scales of flow in the *x, y* directions accordingly – Figure 2.1. Let us consider subregion dV_s , lying in the representative region of the flow, dV, in which random parameters h, h_t, h_x, h_y are changed in the intervals $(h; h + dh)$,$(h_t; h_t + dh_t)$, $(h_x;\ h_x + dh_x)$,$(h_y; h_y + dh_y)$ accordingly. In common case the subregion dV_s is a multiply connected domain. The volume of this subregion is given by

$$dV_s = dV f_s(h, h_x, h_y, h_t) dh dh_x dh_y dh_t \ .$$

The random amplitude of velocity can be determined by the toting expression $\mathbf{u} = \mathbf{u}(x, y, z/h(x, y.t), t)$ in the volume dV_s:

$$\tilde{\mathbf{u}}(h, t, h, h_x, h_{y,} h_t) = \lim_{dV \to dVs} \frac{1}{dV} \int_{dV} \mathbf{u}(x, y, h, t) dx dy dz \qquad (2.1)$$

where dV is an arbitrary volume enclosed in $dV = L_x L_y dz$ and containing dV_s as a whole.

Obviously, $\tilde{\mathbf{u}}(h, t, h, h_x, h_{y,} h_t)$ is the random function, because it depends on the random parameters. The equations, describing dynamics of random functions, $\tilde{\mathbf{u}} = \tilde{\mathbf{u}}(h,t,h,h_x,h_{y},h_t)$, immediately follow from the hydrodynamic equations of viscous fluid and its are adduced below in subsection 2.3. Statistical moment of an order m of the random function $\tilde{\mathbf{u}}(h,t,h,h_x,h_{y},h_t)$ are determined as follows

$$\overline{\tilde{u}}^m(z,t) = \int \tilde{u}^m(h,t,h,h_x,h_y,h_t)\, f_s(h,h_x,h_y,h_t)\,dh\,dh_x\,dh_y\,dh_t \tag{2.2}$$

Thus, in the given theory the mean velocity (and any other mean value) is determined in two steps. On a first steps accordingly to equation (2.1) the random functions $\tilde{u}(h,t,h,h_x,h_{y},h_t)$ are calculated, on the second step accordingly to the equation (2.2) the mean values $\overline{\tilde{u}}^m(z,t)$ are calculated. The similar algorithm has been proposed by Trunev & Fomin [55], and Trunev [56] for the problem of impingement erosion.

The transformation (2.1) can also be developed for the turbulent flows over rough surfaces. One can assume, that the rough surface can be described by a function $z = r(x,y)$ and that the dynamic roughness surface presented as $h(x,y,t) = r(x,y) + \tilde{h}(x,y,t)$, where $\tilde{h}(x,y,t)$ is the thickness of viscous sublayer. Then, in a case of smooth surface we have at $z = 0$: $\mathbf{u} = 0$, and in a case of a rough surface at $z = r$: $\mathbf{u} = 0$. Therefore the factor of static roughness enters in the model as a random parameter. The basic formulas of transformation of flow parameters in the surface layer over a rough surface are given by

$$\tilde{\mathbf{u}}(h,t,r,h,h_x,h_{y},h_t) = \lim_{dV \to dV_s} \frac{1}{dV} \int_{dV} \mathbf{u}(x,y,h,t)\,dx\,dy\,dz \tag{2.3}$$

$$\overline{\tilde{u}}_i^m(z,t) = \int \tilde{u}_i^m(h,t,r,h,h_x,h_y,h_t)\, f_s(r,h,h_x,h_y,h_t)\,dr\,dh\,dh_x\,dh_y\,dh_t$$

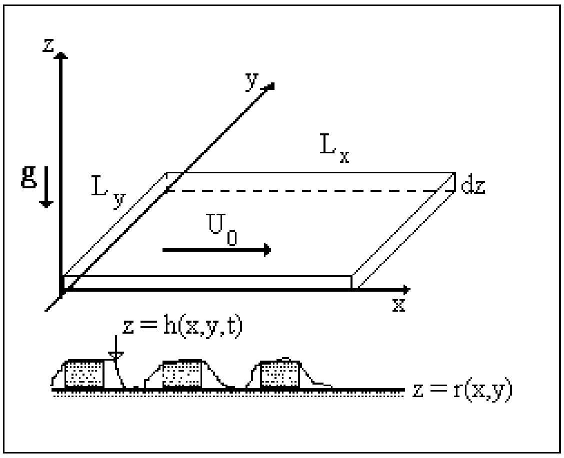

Figure 2.1: Selection of coordinate system at the description of turbulent flows

Let us consider two examples. If $\tilde{u} = u_* \ln(z/r)$, then the mean velocity, calculated on second equation (2.3) can be written as follows

$$\overline{\tilde{u}}(z) = \int \tilde{u}(z/r) f_s(r)dr = u_* \ln(z/r_*) \,,$$

where $r_* = \exp(\int \ln(r) f_s(r)dr)$. Assuming, that $\tilde{u} = u_* \ln(z/r)$ is some solution of the Navier-Stokes equations one can discover, that $\langle u \rangle = \overline{\tilde{u}}(z)$ is also the solution of these equations. If the random amplitude of velocity is $\tilde{u} = u_*(z/r)^b$ and this function is the solution of Navier-Stokes equations, then the mean velocity profile is also the solution $\langle u \rangle = \overline{\tilde{u}}(z) = u_*(z/r_*)^b$ at fixed value of roughness parameter: $r = r_* = \left(\int r^{-b} f_s(r)dr\right)^b$.

The main problem on this way is how to estimate the multiple density of a probability distribution function $f_s = f_s(r,h,h_x,h_y,h_t)$? Nevertheless, for the solutions presented by the logarithmic function one can suppose that

$$\overline{\tilde{u}} = \int \tilde{u}(h_1,t,r,h,h_x,h_y,h_t) f_s(r,h,h_x,h_y,h_t) drdhdh_x dh_y dh_t = \\ = \tilde{u}(z_1/h_*, r_*, h_x^*, h_y^*, h_t^*)$$

where the parameters with stars can be estimated from the experimental data or calculated from the theory of turbulence. Practically the roughness parameter r_* should be given as an input value and all another parameters can be calculated from the similarity theory considered in sections 2.6-2.7.

2.2. Transformation of hydrodynamic equations

2.2.1. Input equations

We shall consider an air flow containing a scalar impurity. Air is assumed as a viscous, heat-conducting, incompressible gas in a rather slow turbulent motion. It is well known fact that the surface layer comprises one-tenth of the planetary boundary layer, in which the earth's rotation effect can be neglected (see Arya [18]). Thus, the model of the turbulent flow in the atmospheric surface layer can be written as follows:

$$\nabla . \mathbf{u} = 0 \tag{2.4}$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}.\nabla)\mathbf{u} + \frac{\nabla p}{r_0} = n\nabla^2 \mathbf{u} + \frac{\mathbf{g}}{r_0}(r - r_0)$$

$$\frac{\partial T}{\partial t} + (\mathbf{u}.\nabla)T = \frac{n}{\mathbf{Pr}}\nabla^2 T$$

$$\frac{\partial f}{\partial t} + (\mathbf{u}.\nabla)f = \frac{n}{\mathbf{Sc}}\nabla^2 f$$

where r is the air density, $\mathbf{u} = (u, \mathrm{v}, w)$ is the flow velocity vector, n is the kinematics viscosity, p is the pressure with the exception of the hydrostatic atmospheric pressure, $\mathbf{g}$ is the gravity acceleration vector, r_0 is the equilibrium density, T is the temperature, $\mathbf{Pr}$ is the Prandtl number, f is the mass concentration of an impurity, $\mathrm{Sc} = n / D$ is the Schmidt number, D is the molecular diffusion coefficient. The hydrostatic equation and the standard Boussinesq approximation for the density fluctuations are given by

$$\nabla p_0 = \mathbf{g} r_0 (p_0, T_0), \quad r - r_0 = -r_0 b (T - T_0)\,, \qquad (2.5)$$

where $b = -r^{-1}(\partial r / \partial T)_p$ is the coefficient of expansion, $b = 1/T$ for the perfect gases.

The coordinates system should be determined in such a way that the Z-axis is directed opposite to the vector of the gravity acceleration. The relief of a ground surface is given by the equation $z = r(x, y)$ - see Figure 2.1.

Boundary conditions for the flow parameters are set on the ground surface and on the top of boundary layer are set as follows:

$$z = r(x, y){:} \quad \mathrm{u} = 0, \quad T = T_g, \quad f = f_g \qquad (2.6)$$

$$z = H{:} \quad \mathbf{u} = U_0 (1{,}0{,}0)\,, \quad T = T_0, \quad f = f_0\,.$$

where T_g is the surface temperature, f_g is the impurity concentration over the ground, H is the boundary layer height, U_0 is the wind velocity at the height $z = H$, T_0, f_0 are the temperature and the impurity concentration at the height $z = H$ respectively.

The boundary conditions (2.6) are spatial homogeneous on horizontal coordinates. This simplification is accepted only at a derivation of the main equations of turbulent flows. Hereinafter the boundary conditions will vary as required, to result them in conformity with a type of soluble problems.

2.2.2. Transformed equations

The transformation (2.3) can be applied to equations (2.4) in the form

$$\tilde{\mathbf{S}}(h, t, r, h, h_x, h_y, h_t) = \lim_{dV \to dV_s} \frac{1}{dV} \int_{dV} \mathbf{S}(x, y, h, t)\,dxdydz$$

where $\tilde{\mathbf{S}} = (\tilde{\mathbf{u}}, \tilde{p}, \tilde{T}, \tilde{f}\,)$, $\mathbf{S} = (\mathbf{u}, p, T, f\,)$. The equations for random functions $\tilde{\mathbf{S}} = (\tilde{\mathbf{u}}, \tilde{p}, \tilde{T}, \tilde{f}\,)$ can be obtained directly from system (2.4) recorded in curvilinear non-stationary coordinates (x, y, h, t), where $h = z / h(x, y, t)$. The transition to curvilinear non-stationary coordinates system in hydrodynamic equations is explicitly described by [59-60] and other. Following Pulliam&Steger [59] equations (2.4) can be presented in the form:

$$\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial}{\partial x}(\mathbf{E} - \mathbf{E}_v) + \frac{\partial}{\partial y}(\mathbf{F} - \mathbf{F}_v) + J\frac{\partial}{\partial h}(\mathbf{G} - \mathbf{G}_v) + J\left[h_t \mathbf{Q} + h_x(\mathbf{E} - \mathbf{E}_v) + h_y(\mathbf{F} - \mathbf{F}_v)\right] = \mathbf{B} \qquad (2.7)$$

where J is the Jacobian of transformation, $J = h^{-1} \neq 0,\ J^{-1} = h \neq 0$,

$$\mathbf{Q} = \begin{pmatrix} r_0 \\ r_0 u \\ r_0 \mathrm{v} \\ r_0 w \\ T \\ f \end{pmatrix},\quad \mathbf{E} = \begin{pmatrix} r_0 u \\ r_0 u^2 + p \\ r_0 \mathrm{v} u \\ r_0 w u \\ uT \\ uf \end{pmatrix},\quad \mathbf{F} = \begin{pmatrix} r_0 \mathrm{v} \\ r_0 u \mathrm{v} \\ r_0 \mathrm{v}^2 + p \\ r_0 w \mathrm{v} \\ \mathrm{v} T \\ \mathrm{v} f \end{pmatrix},\quad \mathbf{G} = \begin{pmatrix} r_0 W \\ r_0 uW - h h_x p \\ r_0 \mathrm{v} W - h h_y p \\ r_0 wW + p \\ WT \\ Wf \end{pmatrix},$$

$$\mathbf{E}_v = \begin{pmatrix} 0 \\ t_{xx} \\ t_{yx} \\ t_{zx} \\ n\mathbf{Pr}^{-1} T_x \\ Df_x \end{pmatrix},\quad \mathbf{F}_v = \begin{pmatrix} 0 \\ t_{xy} \\ t_{yy} \\ t_{zy} \\ n\mathbf{Pr}^{-1} T_y \\ Df_y \end{pmatrix},\quad \mathbf{G}_v = \sum_i \begin{pmatrix} 0 \\ h_i t_{xi} \\ h_i t_{yi} \\ h_i t_{zi} \\ n\mathbf{Pr}^{-1} h_i T_i \\ Dh_i f_i \end{pmatrix},\quad \mathbf{B} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -g(r - r_0) \\ 0 \\ 0 \end{pmatrix}$$

Here t_{kl} is the tensor of viscous stress, $t_{kl} = m\left(\dfrac{\partial u_l}{\partial x_k} + \dfrac{\partial u_k}{\partial x_l}\right)$, m is the dynamic viscosity, $k, l = 1,2,3$; $h_x = -Jhh_x$, $h_y = -Jhh_y$, $h_z = J$, $W = w - h(h_t + h_x u + h_y \mathrm{v}), i = x, y, z$.

In the curvilinear coordinates system it is necessary to execute replacements in terms with gradients as follows:

$$\frac{\partial}{\partial x_j} \to \frac{\partial}{\partial x_j} + \frac{\partial h}{\partial x_j}\frac{\partial}{\partial h} \quad \text{for } j = 1,2;\ \frac{\partial}{\partial z} \to \frac{1}{h}\frac{\partial}{\partial h}.$$

The equations (2.7) in a common case look rather cumbersome and here are not considered. The transformation of the Navier-Stokes model to the non-stationary curvilinear coordinate system is only the convenient method to allocate some thin effect of rough surface in the viscous flow. The turbulent flow over a smooth surface can be considered as the limiting case of flow over a rough surface, when the influence of viscous sublayer exceeds effect of the static roughness.

At first sight it seems that the transition to the new coordinate system (x, y, h, t) is connected with a selection of dynamic roughness surface, and thus it can't be defined as the univalent transformation. Nevertheless, as it will be shown hereinafter, the requirements imposed at calculation of the mean velocity profile allow us to define the mean parameters h_*, h_x^*, h_y^*, h_t^* accurate to an undefined factor, which connects to the scale of the turbulent boundary layer.

2.3 Turbulent boundary layer equations

Let us consider the special type of solution of transformed equations (2.7) which depends on time and normal variable h as it often supposed in the turbulent boundary layer theory. Thus put

$$\frac{\partial}{\partial x}(\mathbf{E}-\mathbf{E}_v)=\frac{\partial}{\partial y}(\mathbf{F}-\mathbf{F}_v)=0$$

in the left part of (2.7). In this case eq. (2.7) can be presented in the form

$$\frac{\partial \mathbf{Q}}{\partial t}+J\frac{\partial}{\partial h}(\mathbf{G}-\mathbf{G}_v)+J\left|h_t\mathbf{Q}+h_x(\mathbf{E}-\mathbf{E}_v)+h_y(\mathbf{F}-\mathbf{F}_v)\right|=\mathbf{B} \qquad (2.8)$$

To derive the turbulent boundary layer model one can apply the averaging operator in the form (2.3) with an arbitrary averaging volume dV to equation (2.8) to conserve the commutative properties of the averaging operator with the space and time differential operators. Then one can consider the limit of all terms of the averaged equation at $dV \to dV_s$. At this step the theorem about two limits of the continuous function can be used (since the differential operators can be considered as some limits). Finally we have

$$\frac{\partial \tilde{\mathbf{Q}}}{\partial t}+J\frac{\partial}{\partial h}(\tilde{\mathbf{G}}-\tilde{\mathbf{G}}_v)+J\left[h_t\tilde{\mathbf{Q}}+h_x(\tilde{\mathbf{E}}-\tilde{\mathbf{E}}_v)+h_y(\tilde{\mathbf{F}}-\tilde{\mathbf{F}}_v)\right]=\tilde{\mathbf{B}} \qquad (2.9)$$

where parameters with tilde are defined similar to $\tilde{\mathbf{S}}=(\tilde{\mathbf{u}},\tilde{p},\tilde{T},\tilde{C})$. If $\tilde{\mathbf{S}}=(\tilde{\mathbf{u}},\tilde{p},\tilde{T},\tilde{C})$ is the solution of the transformed Navier-Stokes equations (2.8) in any sense, then we have the turbulence model closures automatically as follows:

$$\lim_{dV\to dV_s}\frac{1}{dV}\int_{dV}u_i(x,y,h,t)u_k(x,y,h,t)dxdydz=\tilde{u}_i\tilde{u}_k+qd_{ik}$$

$$\lim_{dV\to dV_s}\frac{1}{dV}\int_{dV}\mathbf{u}(x,y,h,t)T(x,y,h,t)dxdydz=\tilde{\mathbf{u}}\tilde{T}$$

$$\lim_{dV\to dV_s}\frac{1}{dV}\int_{dV}\mathbf{u}(x,y,h,t)f(x,y,h,t)dxdydz=\tilde{\mathbf{u}}\tilde{f}$$

Here $3q/2$ is the kinetic energy of turbulent fluctuations in the small volume dV_s, d_{ik} is the Kronecker delta: $d_{ik}=0$ for $i\neq k$, $d_{ik}=1$ for $i=k$.

Therefore (2.8) and (2.9) are the identical equations. The dissipative terms in (2.9) can be written as follows

$$\tilde{\mathbf{E}}_v=-\frac{h}{h}\begin{pmatrix}0\\ 2mh_x\tilde{u}_h\\ mh_y\tilde{u}_h+mh_x\tilde{v}_h\\ mh_x\tilde{w}_h-m\tilde{u}_h/h\\ n\mathbf{Pr}^{-1}h_x\tilde{T}_h\\ Dh_x\tilde{f}_h\end{pmatrix},\quad \tilde{\mathbf{F}}_v=-\frac{h}{h}\begin{pmatrix}0\\ mh_x\tilde{v}_h+mh_y\tilde{u}_h\\ 2mh_y\tilde{v}_h\\ mh_y\tilde{w}_h-m\tilde{v}_h/h\\ n\mathbf{Pr}^{-1}h_y\tilde{T}_h\\ Dh_y\tilde{f}_h\end{pmatrix},\quad \tilde{\mathbf{G}}_v=(1+n^2h^2)\frac{1}{h}\frac{\partial}{\partial h}\begin{pmatrix}0\\ m\tilde{u}\\ m\tilde{v}\\ m\tilde{w}\\ n\mathbf{Pr}^{-1}\tilde{T}\\ D\tilde{f}\end{pmatrix}$$

where $\tilde{u}_h = \partial\tilde{u}/\partial h$, ….

Substituting in (2.9) the expressions of all terms finally we have the dynamic equations for random flow parameters as follows:

$$\frac{\partial\tilde{w}}{\partial h} - h\frac{\partial\Phi}{\partial h} = 0 \qquad (2.10)$$

$$\frac{\partial\tilde{\mathbf{u}}}{\partial t} + \frac{\tilde{W}}{h}\frac{\partial\tilde{\mathbf{u}}}{\partial h} + \frac{\mathbf{N}}{r_0 h}\frac{\partial\tilde{P}}{\partial h} = \frac{n}{h^2}\frac{\partial}{\partial h}(1+n^2h^2)\frac{\partial\tilde{\mathbf{u}}}{\partial h} - \frac{n\,n^2h}{h^2}\frac{\partial\tilde{\mathbf{u}}}{\partial h} + \frac{n\mathbf{N}}{h^2}\frac{\partial\Phi}{\partial h} + \frac{\mathbf{g}}{r_0}(\tilde{r} - r_0)$$

$$\frac{\partial\tilde{T}}{\partial t} + \frac{\tilde{W}}{h}\frac{\partial\tilde{T}}{\partial h} = \frac{n}{\Pr h^2}\frac{\partial}{\partial h}(1+n^2h^2)\frac{\partial\tilde{T}}{\partial h} - \frac{n\,n^2h}{\Pr h^2}\frac{\partial\tilde{T}}{\partial h}$$

$$\frac{\partial\tilde{f}}{\partial t} + \frac{\tilde{W}}{h}\frac{\partial\tilde{f}}{\partial h} = \frac{n}{\mathrm{Sc}h^2}\frac{\partial}{\partial h}(1+n^2h^2)\frac{\partial\tilde{f}}{\partial h} - \frac{n\,n^2h}{\mathrm{Sc}h^2}\frac{\partial\tilde{f}}{\partial h}$$

where $\tilde{W} = \tilde{w} - h\Phi$, $\Phi = h_t + h_x\tilde{u} + h_y\tilde{v}$, $\tilde{P} = \tilde{p} + q$, $n = \sqrt{h_x^2 + h_y^2}$, $\mathbf{N} = (-h\,h_x, -h\,h_y, 1)$.

Note, that the parameters of a dynamic roughness in equations (2.10), are not already the functions of space variables or time. Really, in virtue of transformation (10), the values of these parameters are fixed in intervals from r up to $r + dr$, from h up to $h + dh$, from h_t up to $h_t + dh_t$, from h_x up to $h_x + dh_x$, from h_y up to $h_y + dh_y$. These values, thus, are considered as the random parameters, and the law of their distribution in specific intervals is described by a known function $f_s = f_s(r, h, h_x, h_y, h_t)$.

As we can see from the derived equations (2.10) there are the factors in the second derivatives terms, which depend on a distance up to a rigid surface. It should be noted also, that the equation (2.9) is not in the strong conservation form, as, for example, it is given by Pulliam & Steger [59]. Therefore the number of terms in a square brackets, breaking conservation of this system are chosen in the left part of equations (2.8) and (2.9). Such allocation of non-divergent terms is stipulated by the purposes of modelling of the eddy viscosity, which, in our opinion, arises in a boundary layer from the transformation of a tensor of viscous stresses near the dynamic roughness surface. It is obvious in the case of viscous flow over a rigid rough surface and is connected with an adhesion of a viscous flow to a rigid surface of any configuration. In the turbulent flow over a smooth surface the eddy viscosity is simulated by analogy to a more widespread type of turbulent flows, as in a special case, when $r \to 0$. Thus the eddy viscosity is connected (mathematically) with the transformation of a tensor of viscous stresses to coordinate mapping which brings rigid surface onto coordinate surface.

In this model the Reynolds stress can be calculated as follows

$$t'_{ik}(z,t)=\int r[(\tilde{u}_i-\bar{\bar{u}}_i)(\tilde{u}_k-\bar{\bar{u}}_k)+qd_{ik}]f_s(r,h,h_x,h_y,h_t)drdhdh_xdh_ydh_t$$

Therefore the random function $\tilde{\mathbf{u}}=\tilde{\mathbf{u}}(h,t,r,...)$ gives the main contribution in the non-diagonal components of the Reynolds stress. Now we take it as granted because we haven't any contradictions. Hence, the first assumption of this theory is that the turbulence interaction between the hydrodynamic fields can be described with the solutions $\tilde{\mathbf{S}}=(\tilde{\mathbf{u}},\tilde{p},\tilde{T},\tilde{C})$ as well as with the solutions $\mathbf{S}=(\mathbf{u},p,T,C)$. The second assumption is that it's possible to neglect longitudinal and transversal gradients of flow parameters in a comparison with gradients across a boundary layer, at least for steady turbulent flow.

For the diffusion equation it is possible to derive the boundary layer model by the simplified way. Let us suppose that in the last equation (2.4) $C=C(h(x,y,z,t),t)$, then we have

$$\frac{\partial f}{\partial t}+(\mathbf{u}.\nabla)f-\frac{n}{\mathbf{Sc}}\nabla^2 f=\frac{\partial f}{\partial t}+(h_t+\mathbf{u}.\nabla h)\frac{\partial f}{\partial h}-D(\nabla h)^2\frac{\partial^2 f}{\partial h^2}-D\nabla^2 h\frac{\partial f}{\partial h}=0$$

In partial case when $h=z/h(x,y,t)$ thus

$$\nabla h=h^{-1}(-hh_x,-hh_y,1);\quad \nabla^2 h=-hh^{-1}\nabla^2 h+2h(h^{-1}\nabla h)^2,$$

And therefore the last equation can be written as

$$\frac{\partial f}{\partial t}+\frac{W}{h}\frac{\partial f}{\partial h}=\frac{D}{h^2}(1+n^2h^2)\frac{\partial^2 f}{\partial h^2}+\frac{2Dn^2h}{h^2}\frac{\partial f}{\partial h}-\frac{Dh\nabla^2 h}{h}\frac{\partial f}{\partial h}$$

This equation can be transformed to the form of the last equation (2.10). According to definition

$$\tilde{f}(h,t,r,h,h_x,h_y,h_t)=\lim_{dV\to dVs}\frac{1}{dV}\int_{dV} f(h,t)dxdydz,$$

Using the identity $h^{-1}\nabla^2 h=\nabla(h^{-1}\nabla h)+h^{-2}(\nabla h)^2$, and averaging all terms, finally we have

$$\frac{\partial\tilde{f}}{\partial t}+\frac{\tilde{W}}{h}\frac{\partial\tilde{f}}{\partial h}=\frac{D}{h^2}(1+n^2h^2)\frac{\partial^2\tilde{f}}{\partial h^2}+\frac{Dn^2h}{h^2}\frac{\partial\tilde{f}}{\partial h}-\lim_{dV\to dV_s}\frac{1}{dV}\int dzDh\frac{\partial f}{\partial h}\int_{\Delta S}\nabla(h^{-1}\nabla h)dxdy$$

Where $\Delta S=L_xL_y$. But the last term is annulled if region $\Delta S=L_xL_y$ is large enough (the divergence theorem). Therefore we have an equation

$$\frac{\partial\tilde{C}}{\partial t}+\frac{\tilde{W}}{h}\frac{\partial\tilde{C}}{\partial h}=\frac{D}{h^2}(1+n^2h^2)\frac{\partial^2\tilde{C}}{\partial h^2}+\frac{Dn^2h}{h^2}\frac{\partial\tilde{C}}{\partial h}=\frac{n}{\mathrm{Sc}h^2}\frac{\partial}{\partial h}(1+n^2h^2)\frac{\partial\tilde{C}}{\partial h}-\frac{nn^2h}{\mathrm{Sc}h^2}\frac{\partial\tilde{C}}{\partial h}$$

which is identical to the last equation (2.10).

The hydrodynamic part of the system (2.10) can be transformed to a form convenient for integration. For this purpose one can multiple the second equation (2.10) by scalar way on vectors

$$\mathbf{N},\ \mathbf{N_1} = (h_x, h_y, 0), \mathbf{N_2} = (h_y, -h_x, 0)$$

Expressing the pressure gradient through other parameters one can derive

$$\frac{\partial \tilde{P}}{\partial h} = \frac{2nr_0}{h}\frac{\partial \Phi}{\partial h} - \frac{gh}{\mathbf{N}^2}(\tilde{r} - r_0) - \frac{hr_0}{\mathbf{N}^2}\frac{\partial \tilde{W}}{\partial t} \tag{2.11}$$

$$\frac{\partial \Phi}{\partial t} + \frac{\tilde{W}}{h}\frac{\partial \Phi}{\partial h} - \frac{n^2 h}{r_0 h}\frac{\partial \tilde{P}}{\partial h} = \frac{n}{h^2}\frac{\partial}{\partial h}(1 + n^2 h^2)\frac{\partial \Phi}{\partial h} - \frac{2nn^2 h}{h^2}\frac{\partial \Phi}{\partial h}$$

$$\frac{\partial \Psi}{\partial t} + \frac{\tilde{W}}{h}\frac{\partial \Psi}{\partial h} = \frac{n}{h^2}\frac{\partial}{\partial h}(1 + n^2 h^2)\frac{\partial \Psi}{\partial h} - \frac{nn^2 h}{h^2}\frac{\partial \Psi}{\partial h}$$

where $\mathbf{N}^2 = 1 + n^2 h^2$, $\Psi = h_y u - h_x \mathrm{v}$.

The system (2.11) can be closed using the continuity equation, which is represented for this purpose as follows

$$\frac{\partial \tilde{W}}{\partial h} + \Phi = 0 \tag{2.11, a}$$

Let us consider some similarity properties of the system (2.11-2.11,a) in case of the steady turbulent flow. Put

$$\partial \Phi / \partial t = \partial \Psi / \partial t = \partial \tilde{W} / \partial t = 0$$

in (2.11). Then the dimensionless components of a flow velocity and their combination can be written as follows

$$u^+ = \tilde{u} / u_*, \mathrm{v}^+ = \tilde{\mathrm{v}} / u_*, w^+ = \tilde{w} / u_*,$$

$$y = \Psi / nu_* = u^+ \sin a - \mathrm{v}^+ \cos a\ ,$$

$$j = \Phi / nu_* = u^+ \cos a + \mathrm{v}^+ \sin a + w_0{}^+,$$

$$c = jx - \tilde{w}^+,$$

where $u_* = u_t = \sqrt{t_w / r}$ is the friction velocity, t_w is the wall shear stress, $x = z / l$, $l = h / n, a = \mathrm{arctan}(h_y / h_x), w_0^+ = h_t / nu_*$, $w_0 = h_t / n$ is the second scale of velocity in the turbulent boundary layer.

Substituting a gradient of pressure from the first equation (2.11) in second one, expressing the buoyancy force through the temperature according to approximation (2.5) and using dimensionless variables finally we have:

$$\frac{dc}{dx} = j \tag{2.12}$$

$$(1+x^2)\frac{d^2 j}{dx^2}+(l^+ c+2x)\frac{dj}{dx}+\frac{x l^{+2} BT^+}{1+x^2}=0$$

$$(1+x^2)\frac{d^2 y}{dx^2}+(l^+ c+x)\frac{dy}{dx}=0$$

where $l^+ = l n / u_*$, $T^+ = (T_g - \tilde{T}) / T_*$ is the dimensionless temperature, $B = -gb\, T_* n / u_*^3$, $T_* = q_H / (r\, c_p u_*)$ is the turbulent scale of temperature, q_H is the heat flux from the ground to the air, c_p is the specific heat at the constant pressure of the gas.

It's obvious, that the solutions of the equations system (2.12) are not evidently dependent on the choice of the scale h, and depend on the combinations of random parameters

$$l = h/n, a = \arctan(h_y / h_x), w_0^+ = h_t / nu_* .$$

Thus, the turbulent steady flow equations (2.12) are applicable to the arbitrary scales flows, including the atmospheric flows. This property is also applied to the unsteady flows, if the dimensionless time is determined as follows $t = u_* t / l$. For example, the equation for the contravariant component of velocity in common case can be written as

$$\frac{\partial^2 W^+}{\partial x \partial t}+W^+\frac{\partial^2 W^+}{\partial x^2}-\frac{1}{l^+}\frac{\partial}{\partial x}(1+x^2)\frac{\partial^2 W^+}{\partial x^2}=\frac{x}{1+x^2}\frac{\partial W^+}{\partial t}+\frac{b_0 x(\tilde{r}/r_0-1)}{1+x^2} \tag{2.13}$$

where $W^+ = \tilde{W} / u_*, b_0 = l\, g / u_*^2$.

It should be noted that the hydrodynamic part of the system (2.10), as it follows from (2.11), describes flow, which has all components of velocity. That is the essential difference of this model from the standard models [28-29], which are based on the Navier-Stokes equations averaged accordingly to Reynolds. Using this feature it is possible to simulate the turbulent intensity profiles in the turbulent boundary layer as well as the mean velocity, temperature and impurity concentration.

2.4 Theory of turbulent incompressible flows

2.4.1 Turbulent boundary layer in zero pressure gradient

The turbulent boundary layer over a smooth surface is the well investigated flow [51, 61-62]. We shall consider the steady turbulent flow of incompressible fluid in a boundary layer in zero pressure gradients. Let's assume in equations (2.11)

$$\frac{\partial \Psi}{\partial t}=\frac{\partial \tilde{W}}{\partial t}=\frac{\partial \Phi}{\partial t}=0$$

Then the normal pressure gradient can be written as follows

$$\frac{\partial \tilde{P}}{\partial h} = \frac{2nr}{h}\frac{\partial \Phi}{\partial h}$$

Using this expression, we can transform the equations (2.11) to a form convenient for integration:

$$\frac{\partial \tilde{W}}{\partial h} + \Phi = 0 \tag{2.14}$$

$$\frac{\tilde{W}}{h}\frac{\partial^2 \tilde{W}}{\partial h^2} = \frac{n}{h^2}\frac{\partial}{\partial h}(1+n^2h^2)\frac{\partial^2 \tilde{W}}{\partial h^2}$$

$$\frac{\tilde{W}}{h}\frac{\partial \Psi}{\partial h} = \frac{n}{h^2}\frac{\partial}{\partial h}(1+n^2h^2)\frac{\partial \Psi}{\partial h} - \frac{nn^2h}{h^2}\frac{\partial \Psi}{\partial h}$$

Here the second equation is obtained by a substitution Φ from the continuity equation (1.11, a) in the second equation (1.11). The obtained equations can be investigated in general case. Integrating these equations one time, we have

$$\frac{d\Phi}{dh} = \frac{A_1 \exp[-I(h)]}{1+n^2h^2}, \qquad \frac{d\Psi}{dh} = \frac{A_2 \exp[-I(h)]}{\sqrt{1+n^2h^2}}, \tag{2.15}$$

$$\frac{d^2\tilde{W}}{dh^2} = -\frac{A_1 \exp[-I(h)]}{1+n^2h^2}$$

where A_i are some constants, $I = -\frac{h}{n}\int^h \frac{\tilde{W}\,dh}{1+n^2h^2}$.

The velocity components can be written as

$$\tilde{u} = n^{-2}\left[(\Phi - h_t)h_x + \Psi h_y\right],\ \tilde{\mathrm{v}} = n^{-2}\left[(\Phi - h_t)h_y - \Psi h_x\right],\ \tilde{w} = \tilde{W} + h\Phi,$$

Then using (2.15), one can derive

$$\frac{d\tilde{u}}{dh} = \frac{n^{-2}h_x A_1 e^{-I}}{1+n^2h^2} + \frac{n^{-2}h_y A_2 e^{-I}}{\sqrt{1+n^2h^2}}, \tag{2.16}$$

$$\frac{d\tilde{\mathrm{v}}}{dh} = \frac{n^{-2}h_y A_1 e^{-I}}{1+n^2h^2} - \frac{n^{-2}h_x A_2 e^{-I}}{\sqrt{1+n^2h^2}}, \qquad \frac{d\tilde{w}}{dh} = \frac{A_1 h e^{-I}}{1+n^2h^2}$$

The further analysis of turbulent incompressible flow in a boundary layer will be based on equations (2.16), which will be used for the computing of the mean velocity and turbulent intensity profiles.

2.4.2. Non-linear theory

The second equation in system (2.14) is non-linear. We'll consider the numerical method of its solution. In case of the flow over a smooth surface the boundary conditions can be written as follows

$$h = 0: \quad \tilde{W}(0) = 0, \quad d\tilde{W}/dh = -h_t, \quad d^2\tilde{W}/dh^2 = -A_1, \tag{2.17}$$

where A_1 is the free parameter required to obtain the limited value of the integral $I(h) = -\frac{h}{n}\int_0^h \frac{\tilde{W}dh}{1+n^2h^2}$ for $h \to \infty$. This condition is used to obtain the logarithmic asymptotic of the mean velocity profile. The parameter A_1 has a clear physical sense. because this parameter is directly proportional to the normal pressure gradient on the wall:

$$\frac{\partial \tilde{P}}{\partial h} = \frac{2nr}{h}\frac{\partial \Phi}{\partial h}, \text{ thus } \left.\frac{\partial \tilde{P}}{\partial h}\right|_{h=0} = \frac{2nr}{h}A_1.$$

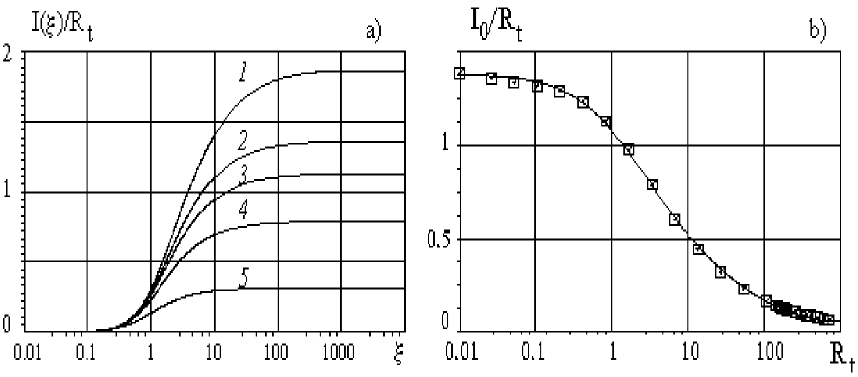

Figure 2.2: a) The normalised function $I(x)/R_t$ computed for various $R_t = -0.83; 0.026; 0.83; 3.32; 26.56$ - the solid lines *1-5* respectively; b) The normalised integral $I_0(R_t)/R_t$ depending on the dynamic roughness parameter for $R_t > 0$

First and second boundary conditions (2.17) can be derived from the definition of $\tilde{W}$ and from the boundary conditions for the viscous flow velocity on a rigid surface.

To minimise the number of the independent random parameters the general solution of the second equation (2.14) can be written as $\tilde{W} = -(h_t/n)c_1(nh, R_t)$, where the universal function c_1 depends on the composition of random parameters (the dynamic roughness Reynolds number):

$$R_t = \frac{hh_t}{n(h_x^2 + h_y^2)} \quad (2.18)$$

and satisfies to the equation

$$(1+x^2)\frac{d^3c_1}{dx^3} + (R_t c_1 + 2x)\frac{d^2c_1}{dx^2} = 0 \quad (2.19)$$

with boundary conditions at

$$x = 0: \quad c_1(0) = 0, \quad dc_1/dx = 1, \quad d^2c_1/dx^2 = a \quad (2.20)$$

where $x = nh$, a is also the free parameter required to obtain the limited value of the integral $I(x)$ at $x \to \infty$. Note, that (2.19) can be derived from the second equation (2.14) and that $a = l^+\tilde{P}_x(0)/2ru_*^2$. Consequently the integral $I(x)$ depends on the composition of random parameters R_t and can be calculated as

$$I(x, R_t) = \int_0^x \frac{R_t c_1(x, R_t)\,dx}{1+x^2} \quad (2.21)$$

The integral (2.21) has been computed in the range $-2.5 \le R_t \le 700$ together with problem (2.19-2.20)). Fourth-order scaled Runge-Kutta algorithms and the shooting method (see [63]) have been used to get the numerical solution. Note that for $R_t = 0$ this problem has the analytical solution:

$$c_1(x) = \int_0^x (1 + a\arctan x)dx\ .$$

In this case $I(x) = 0$, hence one can suggest that function

$$\hat{I}(x) = \lim_{R_t \to 0} I(x)/R_t = \int_0^x \frac{c_1(x)\,dx}{1+x^2}$$

has a limited value for $x \to \infty$. It is possible if only $a = -2/p$, thus

$$c_1(x) = \int_0^x (1 - \frac{2}{p}\arctan x)dx$$

This solution has been used as the initial position in the shooting method.

The normalized function $I(x)/R_t$ is shown in Figure 2.2,a for various $R_t = -0.83; 0.026; 0.83; 3.32; 26.56$ - solid lines ***1-5*** respectively. As it is shown the function $I(x)/R_t$ is simple and smooth function such $\arctan(x)$. The calculated limited value $I_0(R_t) = \lim_{x\to\infty} I(x, R_t)$ is shown in Figure 2.2,b by the symbols together with the approximated line. To simplifier the numerical modelling of the mean velocity profile over a rough surface the function $I(x, R_t)$ has been approximated as

$$I(x, R_t) \cong \frac{2}{p} I_0(R_t) \arctan[(0.4 + 0.02 R_t^{3/4}) x] \qquad (2.22)$$

where $I_0(R_t)$ is given by

$$I_0(R_t) / R_t = 1.38 - 1.13 \arctan[0.4 \ln(1 + R_t^{\,q})],$$

$$q = \begin{cases} 1, & 0 \le R_t \le 100 \\ 1 - (1.5 R_t - 150) 10^{-4}, & 100 < R_t \le 700 \end{cases}$$

Finally note that for the negative value of the parameter R_t in the range $R_t < -2.5$ the numerical procedure becomes unstable one. In this case the value $I_0(R_t) = \lim_{x \to \infty} I(x, R_t)$ increases considerably with the small decreasing of the dynamic roughness parameter R_t. Since this branch of the integral $I_0(R_t)$ will not be used in the analysis, therefore data for the negative value $R_t < 0$ is not presented in Figure 2.2, b and has been neglected in approximation formula (2.22).

2.4.3. Mean velocity profile in turbulent flow over smooth surface

The turbulent boundary layer over a smooth surface is the best example for the theoretical consideration and modelling according to the model (2.16). In this case the streamwise velocity gradient can be written in the standard form using the inner layer variables $z^+ = z u_t / n$, $u^+ = \tilde{u} / u_t$, and boundary condition for the mean velocity gradient on the smooth wall:

$$\text{at } z^+ \to 0: du^+ / dz^+ \to 1 .$$

Besides one can require, that at the great distance from the wall the profile of mean velocity is described by the logarithmic function, i.e.:

$$\text{at } z^+ \to \infty \quad du^+ / dz^+ \to 1 / k\, z^+ ,$$

where k is the Karman constant (we use the parameters with stars instead of the random parameters according to subsection 2.1, thus it's the reason why the Karman constant has been taken). Finally we have got for the streamwise velocity gradient

$$\frac{du^+}{dz^+} = \frac{A e^{-I}}{1 + (z^+ / l^{\,+})^2} + \frac{e^{I_0 - I}}{k l^{\,+} \sqrt{1 + (z^+ / l^{\,+})^2}} \qquad (2.23)$$

where $A = 1 - e^{I_0} / k l^{\,+}$, $l^{\,+} = h u_t / nn$.

The first term in the right part (2.23) has the essential value mainly close to the wall (if $A \ne 0$) and the second one gives the main contribution in the logarithmic layer. To derive the mean velocity profile we should firstly define the parameter $A = 1 - e^{I_0} / k l^{\,+}$. Note, from first equation (2.16) and (2.23) it follows that

$$A = \cos^2 a \, \frac{du^+(0)}{dz^+} + \cos a \, \sin a \, \frac{d\,\mathrm{v}^+(0)}{dz^+}$$

Our suggestion about the dynamical roughness structure is that the parameter a fluctuates around the mean value $a = p/2$. This structure looks like furrows elongated along of the mean flow stream lines in the viscous sublayer (see, for instance, Cantwell, Coles & Dimotakis [64]) where the visualisation of the coherent structure in the turbulent boundary layer is presented, and subsection 2.5 here).

Thus for the mean flow $A = 1 - e^{I_0} / k\, l^+ = 0$, then the length scale $l^+ = hu_t / nn$ can be found as the solution of the next equation

$$k = w_0^+ \exp[I_0(R_t)] / R_t \tag{2.24}$$

where $R_t = l^+ w_0 / u_t$, $w_0 = h_t / n$ is the second scale of the turbulent velocity. For an arbitrary value w_0 the equation (2.24) has two roots or hasn't any roots and only if $dk / dR_t = 0$ this equation has one root. Hence for the uniqueness of the mean velocity profile should be done

$$\frac{1}{w_0^+} \frac{dk}{dR_t} = \frac{e^{I_0}}{R_t} \frac{dI_0}{dR_t} - \frac{e^{I_0}}{R_t^{\,2}} = 0 \tag{2.25}$$

The numerical solution of the equation (2.25) with $I_0(R_t)$ determined from (1.22) gives $R_t = R_t^* \approx 1.22$ and therefore the predicted values of the turbulent theory constants are given by (for $k = 0.41$)

$$w_*^+ = kR_t^* \exp(-I_0) = 0.14\,, \quad l_0^+ = R_t^* / w_*^+ = 8.71 \tag{2.26}$$

The fundamental parameter of length for the turbulent boundary layer is defined from here: $l_0 = l_0^+ n / u_* \approx 8.71 n / u_*$, that almost coincides with the peak of turbulence production, obtained by Klebanoff [49] and Laufer [50]. The fundamental scale of length in the turbulent boundary layer, determined as $l_0^+ = R_t^* / w_*^+ = 8.71$ also coincides with the peak of the frequency diagram of instantaneous thickness of a viscous sublayer [62] - see Fig. 2.3.

If $A = 0$ then $l^+ = l_0^+$ in (2.23) and this equation can be presented in the form:

$$\frac{du^+}{dz^+} = \frac{e^{I_0 - I}}{k\, l_0^+ \sqrt{1 + (z^+ / l_0^+)^2}} \quad , \tag{2.27}$$

where $I(z^+ / l_0^+, R_t^*) = \int_0^x \frac{R_t^* c_1(x, R_t^*)\, dx}{1 + x^2}$.

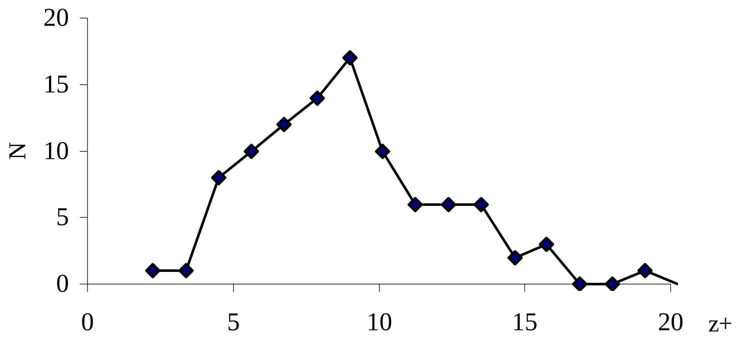

Figure 2.3: Frequency diagram of instantaneous thickness of a viscous sublayer in the turbulent boundary layer [62]

Integrated the first equation (2.27) we have:

$$u^+ = \int_0^{z^+} \frac{e^{I_0 - I} dz^+}{k\, l_0^+ \sqrt{1 + (z^+ / l_0^+)^2}} = \frac{1}{k}\int_0^x \frac{(e^{I_0-I} - 1)dx}{\sqrt{1+x^2}} + \frac{1}{k}\int_0^x \frac{dx}{\sqrt{1+x^2}} =$$

$$= \frac{1}{k}\int_0^x \frac{(e^{I_0-I} - 1)dx}{\sqrt{1+x^2}} + \frac{1}{k}\ln\left(\frac{z^+}{l_0^+} + \sqrt{1 + \left(\frac{z^+}{l_0^+}\right)^2}\right)$$

The standard logarithmic profile can be derived from here at $z^+ >> l_0^+$:

$$u^+ = \frac{1}{k}\ln z^+ + c_0, \quad c_0 = \frac{1}{k}\int_0^\infty \frac{e^{I_0-I} - 1}{\sqrt{1+x^2}} dx - \frac{1}{k}\ln\frac{l_0^+}{2}. \qquad (2.28)$$

Therefore, with the given constant k another constant of the mean velocity logarithmic profile can be calculated from the second equation (2.28). It gives $c_0 = 5.015$ for $k = 0.41$. The velocity profile calculated with (2.27) for $k = 0.41$; $l_0^+ = 8.71$ is shown in Figure 2.4, a by the solid line (*1*). For this profile the shooting parameter in (2.20) is estimated as $a = -2/p - 0.27 \approx 0.9066$. The predicted profile (*1*) has been compared with the mean velocity profile computed on the model of the transitional layer proposed by Van Driest [65] - the solid line (*2*). As explained by Cebeci & Bradshaw [51] the Van Driest's model can be written in the form

$$\frac{du}{dz} = \frac{(-<u'w'>)^{1/2}}{k\, z[1-\exp(-z/l_d)]}, \qquad -<u'w'> + n\frac{du}{dz} = u_t^{\,2} \qquad (2.29)$$

Here l_d is the damping length, $l_d = 26n / u_t$. The model (2.29) is based on the Prandtl theory of mixing length. Thus, this model depends on two parameters: the Karman constant and the damping length.

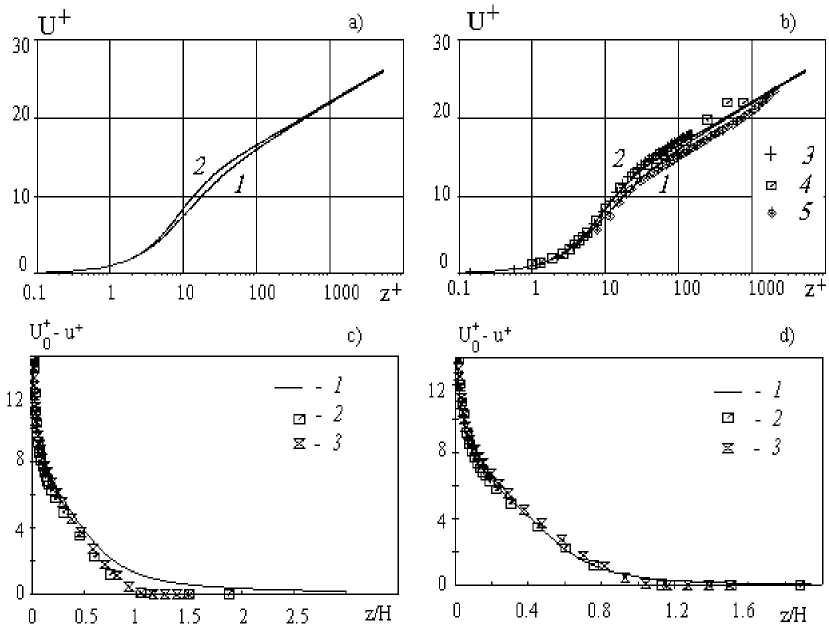

Figure 2.4: a) Mean velocity profiles in the turbulent boundary layer for inner region calculated on eq. (2.27) - (*1*), and with Van Driest model (2.29) -(*2*); b) comparison of computed profiles (*1*-*2*) with DNS data by Kuroda et al [66] - (*3*), and with experimental data by Nagano et al [68] and Smith [69] - (*5*); c) mean velocity defect low in the outer region: solid line *1* is calculated on eq. (2.34) for $V_* = 0.4$ and experimental data (*2*,*3*) by Nagano et al [68] obtained at $x = 0.525м; 1.125м$ accordingly; d) mean velocity defect low in the outer region: solid line *1* is calculated on eq. (2.35) for $V_* = 0.27; e = 0.8$ with experimental data (*2*,*3*) by Nagano et al [68]

The profile computed on (1.29) coincides with the predicted profile *1* in the viscous sublayer and in the logarithmic layer but differs a bit in the transitional layer (see Figure 2.4, a). This difference can be explained by the pressure gradient effect. Figure 2.4,b demonstrates the comparison of both profiles (*1*,*2*) with several data bases: *3* - the direct numerical simulation of the turbulent flow in the two-dimensional channel ($\mathrm{Re} = 2980$) by Kuroda *et al* [66]; *4* - the turbulent boundary layer in zero pressure gradient (the Reynolds number based on the momentum thickness $\mathrm{Re}_q = 1040$) by Nagano *et al* [67,68] and *5* - the turbulent boundary layer mean velocity profile ($\mathrm{Re} = 13052$) presented by Smith [69]. Note that both profiles (*1,2*) well correlated with computed and experimental data.

In the upper layer for $z^+ \geq H^+ = Hu_*/n$ the mean velocity profile should be constant in contrast to the logarithmic profile which diverges at $z^+ \to \infty$. As it is well known in the outer region of the turbulent boundary layer the mean velocity profile can be described by the defect low:

$$U^+ = \frac{U_0 - u}{u_t} = -\frac{1}{k}\ln\frac{z}{H} + F_H\left(\frac{z}{H}\right) \tag{2.30}$$

where the universal function $F_H(V)$ has the properties:

$$\lim_{V\to 0}\frac{dF_H}{dV} = 0; \quad \lim_{V\to\infty} F_H(V) = -\frac{1}{k}\ln V \tag{2.31}$$

Here the first condition means that the universal function does not change boundary conditions on a rigid wall. The second condition can be used to obtain the limiting value of the mean velocity $u = U_0$.

One can suggest that the mixed layer turbulence is generated in the same way as the wall turbulence. Then the new dynamic roughness surface can be introduced and the equation system which is similar to (2.16) can be derived. In the case of the mixing layer we can put $I = I_0$ and $l = V_* H$, where V_* is the parameter. Using the new characteristic scale and equation (2.30) the velocity gradient can be written as

$$\frac{du^+}{dV} = \frac{1}{kV} - \frac{1}{kV_*}\left(1+\bar{z}^2\right)^{-1/2} + \frac{1}{kV_*}\frac{\sqrt{1+\bar{z}_0^2}}{(1+\bar{z}^2)} \tag{2.32}$$

where $\bar{z} = (z - z_0)/V_* H = (V - V_0)/V_*$, $z_0 = H/2$ is the middle position of the mixing layer.

Obviously, that the first term in the right (2.32) part corresponds to the logarithmic profile in the inner layer. Therefore the combined mean velocity profile can be written as follows

$$u^+ = u_{in}^+(z^+) - \frac{1}{k}\left(\mathrm{Arsh}(\bar{z}) + \mathrm{Arsh}(\bar{z}_0)\right) + \frac{\sqrt{1+\bar{z}_0^2}}{k}\left(\arctan\bar{z} + \arctan\bar{z}_0\right) \tag{2.33}$$

where $u_{in}^+(z^+)$ is the mean velocity profile in the inner layer, which is given by equation (2.27), $\mathrm{Arsh}(\bar{z}) = \ln(\bar{z} + \sqrt{1+\bar{z}^2})$, $\bar{z}_0 = 1/2V_*$.

Using the asymptotic formula $\mathrm{Arsh}(\bar{z}) = \ln(2\bar{z})$ for $\bar{z} \to \infty$ one can derive from (2.33):

$$U_0^+ = c_0 - \frac{1}{k}\left(\ln\frac{2}{V_* \,\mathrm{Re}_*} + \mathrm{Arsh}(\bar{z}_0)\right) + \frac{\sqrt{1+\bar{z}_0^2}}{k}\left(\frac{p}{2} + \arctan\bar{z}_0\right)$$

where $\mathrm{Re}_* = Hu_t / n$ is the dynamic Reynolds number. Then, subtracting the equation (2.33) from both parts of this expression we have got the mean velocity defect low in the upper layer

$$U_0^+ - u^+ = -\frac{\ln(2V/V_*)}{k} + \frac{\mathrm{Arsh}(\bar{z})}{k} + \frac{\sqrt{1+\bar{z}_0^2}}{k}\left(\frac{p}{2} - \arctan\bar{z}\right) \qquad (2.34)$$

Thus, the defect low (2.34) depends on the parameter V_*, which can precisely be determined, for instance, in case of turbulent flow in a flat channel from the additional condition $u_V^+(1) = 0$. However in the boundary layer this condition seems artificial as well as concept of the external boundary. It can be understood if the mean velocity defect low (2.34) to compare with the experimental data - see Figure 2.4, c. Figure 2.4,c shows that the computed profile (solid line *1*) comes to zero when the parameter $z/H \geq 3$, whereas the experimental data concentrates near zero for $z/H = 1$. It should be noted, that the parameter of the profile *1* is calculated by minimisation of root-mean-square deviation of experimental points from the computational curve. That is reached for $V_* = 0.4$.

To obtain the best correlation with the experimental data, the first condition (2.31) has been changed and. Then the mean velocity defect low can be rewritten as

$$U_0^+ - u^+ = -\frac{\ln(2V/V_*)}{k} + \frac{\mathrm{Arsh}(\bar{z})}{k} + \frac{e\sqrt{1+\bar{z}_0^2}}{k}\left(\frac{p}{2} - \arctan\bar{z}\right) \qquad (2.35)$$

where e is the parameter.

The mean velocity defect low computed on (2.35) (the solid line *1*) and the experimental data by Nagano *et al* (1992) are shown in Figure 2.4,d. As it has been established $V_* = 0.27;\, e = 0.8$. After this correction the mean velocity defect low in the form (2.35) is in a good agreement with the experimental data - see Figure 2.4.d.

It is necessary to take into account the contribution of the upper layer universal function gradient to the mean velocity gradient on the wall, because the first condition (2.31) is broken. For this purpose the expression (2.33) can be modified as follows

$$u^+ = u_{in}^+(z^+) - \frac{1}{k}\left(\mathrm{Arsh}(\bar{z}) + \mathrm{Arsh}(\bar{z}_0)\right) + \frac{e\sqrt{1+\bar{z}_0^2}}{k}\left(\arctan\bar{z} + \arctan\bar{z}_0\right)$$

Differentiating both parts of this expression and calculating the derivative near the wall, we have

$$\frac{du^+}{dz^+} = \frac{du^+_{in}}{dz^+} - \frac{1-e}{kV_* \operatorname{Re}_* \sqrt{1+\bar{z}_0^{\,2}}} = 1 \qquad (2.36)$$

Therefore the contribution of the upper layer universal function gradient to the mean velocity gradient on the smooth wall decreases with the growth of the dynamic Reynolds number. Using (2.36) the mean velocity profile in the turbulent boundary layer can be written as

$$u^+ = e_0 u^+_{in}(z^+) - \frac{e_0}{k}\left(\operatorname{Arsh}(\bar{z}) + \operatorname{Arsh}(\bar{z}_0)\right) + \frac{e\sqrt{1+\bar{z}_0^2}}{k}\left(\arctan \bar{z} + \arctan \bar{z}_0\right) \qquad (2.37)$$

where $e_0 = 1 + (1-e)/kV_* \operatorname{Re}_* \sqrt{1+\bar{z}_0^2} \approx 1 + 0.9/\operatorname{Re}_*$.

The constants of the theory of turbulence should be re-normalized due to (2.36) as follows

$$w_0^+ = e_0 k R_t^* \exp(-I_0) = 0.14 e_0\,, \quad l_0^+ = R_t^* / w_0^+ = 8.71/e_0 \qquad (2.38)$$

It is obvious, that the uncertainty of constants in the turbulent boundary layer on a flat plate is stipulated by the fact, that this flow is two-dimensional one, because it develops from the laminar boundary layer through the transition layer to the developed turbulent layer, down to the verge of separation (see chapter 6).

2.4.4 Streamwise turbulent intensity profiles

The variations of the velocity gradient around the mean value given by the first equation (2.27) are the production of two terms. One of them depends on the variations of the parameter A, which can estimated as $dA = d(1-e^{I_0}/kl^+) = -dw_0^+ e^{I_0}/kR_t^* = -dw_0^+/w_0^+$; another one depends on the fluctuating part of the wall shear stress t_w or boundary condition on the wall - see Figure 2.5. Thus, in common case the velocity gradient can be written as

$$\frac{du^+}{dz^+} = \frac{Ae^{-I}}{1+(z^+/l^+)^2} + \frac{(E-A)e^{-I}}{\sqrt{1+(z^+/l^+)^2}} \qquad (2.39)$$

Boundary condition on the wall: $z^+ \to 0: du^+/dz^+ \to E$ where $E = 1$ for the mean flow.

Therefore the fluctuating streamwise velocity gradient can be written as

$$\frac{ddu^+}{dz^+} = \frac{dAe^{-I}}{1+(z^+/l_0^+)^2} + \frac{dA(dE/dA-1)e^{-I}}{\sqrt{1+(z^+/l_0^+)^2}} \qquad (2.40)$$

Let us suggest that for the considered turbulent flow $dE/dA = const$, then we have:

$$\left\langle (du^+)^2 \right\rangle = \left\langle (dA)^2 \right\rangle f^2(z^+, dE/dA) \qquad (2.41)$$

Here $f(z^+, dE/dA)$ is the solution of the value boundary problem:

$$\frac{df}{dz^+} = \frac{e^{-I}}{1+(z^+/l_0^+)^2} + \frac{(dE/dA-1)e^{-I}}{\sqrt{1+(z^+/l_0^+)^2}} \qquad (2.42)$$

where $f(0) = f(d_0) = 0$ in case of the turbulent boundary layer on the flat plate, and $f(0) = 0,\ f(d_0) = f_0$ for the turbulent flow in the two-dimensional channel.

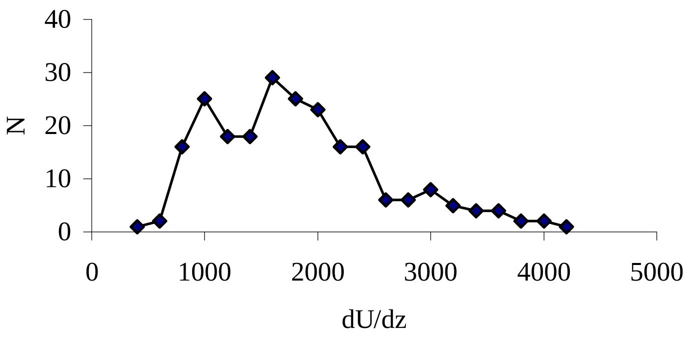

Figure. 2.5: Frequency diagram of the wall shear stress in the turbulent boundary layer [62]. Mean velocity gradient $d\overline{U}/dz = 2000\ c^{-1}$

One constant of the mean-squared value of the fluctuating streamwise velocity profile can be found out from the boundary value problem (2.42) solution and another one can be determined from the experiment. The predicted streamwise turbulent intensity profile, $dU^+ = \sqrt{\langle (dA)^2 \rangle} f(z^+)$ and the profile by Kuroda *et al* [66] computed by the direct numerical simulation of the turbulent flow in the two-dimensional channel for $\mathrm{Re} = 2980$, are shown in Figure 2.6,a. The constants determined for this case are given by $\sqrt{\langle (dA)^2 \rangle} = 0.925,\ dE/dA = 0.444$. Figure 2.6,b illustrates the comparison of the computed streamwise turbulent intensity profile (solid line) with the experimental data by Nagano *et al* [68] (the line with symbols). In this case the best correlation of the computed and experimental data is for $\sqrt{\langle (dA)^2 \rangle} = 0.735,\ dE/dA = 0.56$.

Estimating the contribution of the outer region of the turbulent boundary layer to the streamwise turbulent intensity, one can assume, that the velocity fluctuations in the upper layer depend on the fluctuations of the velocity gradient on the wall. Then the generalized form of the expression (2.37) can be written as

$$u^+ = e_0 u_{in}^+(z^+) - \frac{(E-A)e_0}{k}\left(\mathrm{Arsh}(\bar{z}) + \mathrm{Arsh}(\bar{z}_0)\right) + \frac{(E-A)e\sqrt{1+\bar{z}_0^2}}{k}\left(\arctan\bar{z} + \arctan\bar{z}_0\right) \qquad (2.43)$$

where $u_{in}^{+}(z^{+})$ is given by the equation (2.39). Varying parameters *A, E* in this profile, we have

$$du^{+}=e_0 du_{in}^{+}(z^{+})-\frac{(dE-dA)e_0}{k}\left(\mathrm{Arsh}(\bar z)+\mathrm{Arsh}(\bar z_0)\right)+\frac{(dE-dA)e\sqrt{1+\bar z_0^2}}{k}\left(\arctan\bar z+\arctan\bar z_0\right) \quad (2.44)$$

where $du_{in}^{+}(z^{+})$ is calculated from (2.40). Hence, in this case it is possible to present the streamwise turbulent intensity profile as follows $dU^{+}=\sqrt{\langle (dA)^2\rangle}\,\bar f(z)$, where

$$\bar f(z)=e_0 f(z^{+})-\frac{(dE/dA-1)e_0}{k}\left(\mathrm{Arsh}(\bar z)+\mathrm{Arsh}(\bar z_0)\right)+\frac{(dE/dA-1)e\sqrt{1+\bar z_0^2}}{k}\left(\arctan\bar z+\arctan\bar z_0\right) \quad (2.45)$$

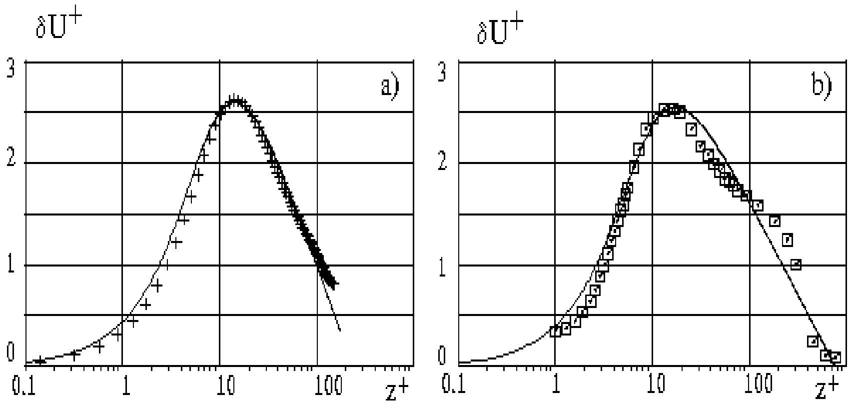

Figure 2.6: a) Comparison of computed streamwise turbulent intensity profile with DNS data by Kuroda *et al* [66], b) experimental data by Nagano *et al* [67, 68]

The streamwise turbulent intensity profile computed on equation (2.45) and the experimental data by Nagano *et al* [68] are shown in Figure 2.7,a. Comparing the computed profiles in Figures 2.6, b and 2.7,a one can conclude that the boundary conditions on the external border for the turbulent intensity profile is differ for this two models. The profile computed on the model (2.45) has zero value together with the first derivative and then at $z > H$ saves zero value, whereas the profile (2.42) approaching to zero has the nonzero decline.

The parameters of the profile (2.45) are given by $\sqrt{\langle (dA)^2\rangle}=0.7,\ dE/dA=0.568$, thus its practically have the same values as for the profile shown in Figure 2.6,b. The fluctuation of the velocity profile declination on the wall are connected with the fluctuation of the shear stress, $dE = dt_W / t_W$. This value can be defined from the data shown in Figures 2.6, 2.7. For the turbulent flow in the flat channel this value is given by $dt_W / t_W = dA(dE/dA) = 0.41$.

The same value approximately we have for the turbulent boundary layer on the flat plate.

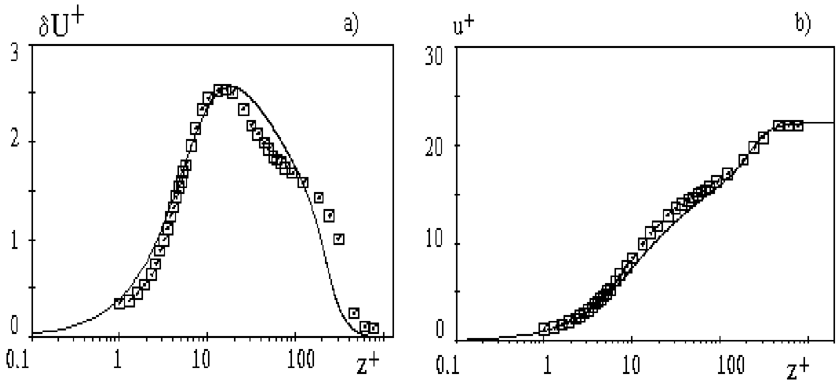

Fig. 2.7: a) The streamwise turbulent intensity profile in the turbulent boundary layer calculated on (2.45) - the solid line, and the experimental data [68]; b) the mean velocity profile in the turbulent boundary layer, calculated on (2.37) for $V_* = 0.27; e = 0.79$, and the experimental data [68] obtained in the cross-section $x = 0.525\,м$

2.4.5. Normal and transversal turbulent intensity profiles

The flow in the turbulent boundary layer is not characterised by significant value of the normal and transversal (parallel to the wall) mean velocity components. Therefore the turbulent fluctuation of velocity in the specified directions can be considered as the random deviations of flow velocity vector from the mean value. The main parameter of these deviations is direct proportional to the dimensionless pressure normal gradient on the wall. Assuming that in the equation system (2.16) $a = p/2$, let us rewrite the last two equations as follows

$$\frac{dw^+}{dx} = x\frac{d\,\mathrm{v}^+}{dx} = \frac{\mathrm{v}_0^+ x e^{-I}}{1+x^2} \qquad (2.46)$$

where $\mathrm{v}_0^+ = aw_0^+$ is the turbulent velocity scale characterised the flow velocity pulsation in the normal and transversal directions. Suggesting that $R_t = R_t^*$, then the turbulent velocity scale can be written through the characteristic length scale, thus $\mathrm{v}_0^+ = aR_t^* / l^+$. At fixed turbulent theory parameters $R_t = R_t^*$, $l^+ = l_0^+$ the solutions of equations (2.46) depend on the parameter a, which is proportional to the dimensionless normal to the wall gradient of pressure, $a = l^+\tilde{P}_x(0)/2ru_*^2$.

Integrating equations (2.46) the velocity components in the inner layer can be written as follows

$$v^+/v_0^+ = \int_0^x \frac{(e^{-I} - e^{-I_0})dx}{1+x^2} + e^{-I_0}\arctan x \quad (2.47)$$

$$w^+/v_0^+ = \int_0^x \frac{(e^{-I} - e^{-I_0})xdx}{1+x^2} + \frac{e^{-I_0}}{2}\ln(1+x^2)$$

It is necessary to add the compensatory functions, which depend on the upper layer variable $\bar{z} = (z - z_0)/V_* H$, to these expressions, thus we have

$$v^+/v_0^+ = \int_0^x \frac{(e^{-I} - e^{-I_0})dx}{1+x^2} + e^{-I_0}\arctan x + e_1(\arctan \bar{z} + \arctan \bar{z}_0)$$

$$w^+/v_0^+ = \int_0^x \frac{(e^{-I} - e^{-I_0})xdx}{1+x^2} + \frac{e^{-I_0}}{2}\ln(1+x^2) + \frac{e_1}{2}\ln\frac{1+\bar{z}^2}{1+\bar{z}_0^2}$$

where parameter e_1 is determined from the boundary conditions:

$$\lim_{z\to\infty} v^+ = v_\infty^+; \quad \lim_{z\to\infty} w^+ = w_\infty^+$$

Therefore we have $e_1 = -e^{-I_0}$ and two additional equations for calculating of the parameters of this problem:

$$\int_0^\infty \frac{(e^{I_0 - I} - 1)dx}{1+x^2} - \arctan \bar{z}_0 = \frac{v_\infty^+ e^{I_0}}{v_0^+} \quad (2.48)$$

$$\int_0^\infty \frac{(e^{I_0 - I} - 1)xdx}{1+x^2} + \frac{1}{2}\ln(1+\bar{z}_0^2) + \ln\frac{V_* \mathrm{Re}_*}{l^+} = \frac{w_\infty^+ e^{I_0}}{v_0^+}$$

If all parameters of the inner and outer layers are fixed, i.e., $R_t = R_t^*, w_0^+ \approx 0.14, l^+ = l_0^+$, then the normal and transversal velocity fluctuations depend only on the parameter $a = l^+\tilde{P}_x(0)/2ru_*^2$. In this case the turbulent intensity of the transversal velocity can be written as follows:

$$d\,v^+ = w_0^+ d\,a[v_{in}^+(z^+) - v_{out}^+(\bar{z})] \quad (2.49)$$

where v_{in}^+ is the function of the inner layer calculated according to the first equation (2.47) at $v_0^+ = 1$. The function of the outer layer is determined as follows

$$v_{out}^+ = e^{I_0}(\arctan \bar{z} + \arctan \bar{z}_0).$$

The transversal turbulent intensity profile computed on (2.49) is shown in the left part of Figure 2.8 together with DNS data by ***Kuroda et al*** (1989). The normal pressure gradient fluctuation parameter has been estimated from these data as $da \approx 1.1 / w_0^+ \approx 7.87$ and consequently $\mathrm{v}_0^+ = w_0^+ da \approx 1.1$.

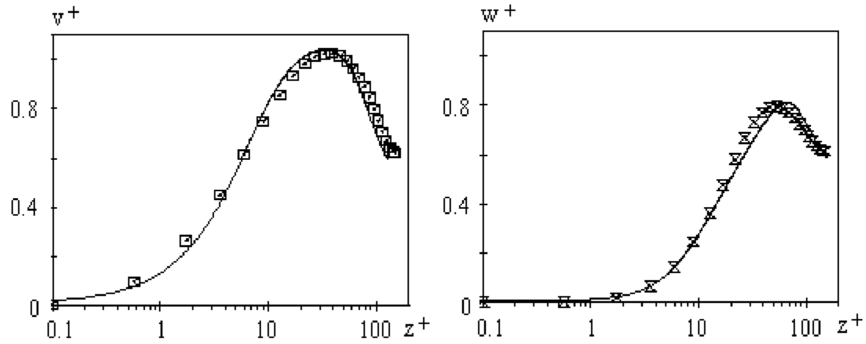

Figure 2.8: Turbulent intensity profiles of transversal velocity components in the turbulent boundary layer in the two-dimensional channel computed on (1.49) - (1.50) - solid lines, and DNS data by Kuroda *et al* [66]

The similar expression for the turbulent intensity of the normal velocity component is given by

$$dw^+ = w_0^+ da[w_{in}^+(z^+) - w_{out}^+(\bar{z})] \tag{2.50}$$

where the function of the inner layer can be calculated on second equation (2.47) at $\mathrm{v}_0^+ = 1$. The outer layer function is determined in this case as follows

$$w_{out}^+ = e^{I_0}[\ln(1+\bar{z}^2) - \ln(1+\bar{z}_0^2) + \arctan\bar{z} + \arctan\bar{z}_0].$$

The normal turbulent intensity profile calculated on (2.50) is shown in the right part of Figure 2.8 with DNS data by Kuroda *et al* [66]. As it has been estimated the turbulent normal velocity scale $\mathrm{v}_0^+ \approx 0.75$, i.e., it is not equal to the transversal velocity scale. This is obviously connected to the influence of dynamic roughness parameters, because in common case $a \neq p/2$. However it is difficult to calculate this influence, so far as we have the non-linear problem.

2.5. Dynamic roughness surface equation

The dynamic roughness surface is connected with the coherent structures in the turbulent boundary layer reported by Kline et al [70]. The review of the coherent structure problem, including the evaluation of the basic structure scales has been submitted by Cantwell [52]. To derive the dynamic roughness surface equation, the common method developed by Trunev *et al* [71-73] can be used. This method was proposed for the simulation of rough surfaces which are formed on the rigid body in the impingement erosion process, including the sputtering of a rigid body by ionic bombardment.

The Reynolds number calculated on the dynamic roughness parameters is given by

$$R_t^* = l_0^+ w_0^+ = \frac{h_t^* h^*}{n(h_x^{*2} + h_y^{*2})}.$$

This statistical relationship between the dynamic roughness parameters corresponds to the special type of the Nave-Stocks equation solution transformation (2.4). Let's consider small variation of the second velocity scale around the mean value $w_0^+ = k\, R_t^* \exp[-I_0(R_t^*)] \approx 0.14$. In this case $A \neq 0$ in the first equation (2.16) and this parameter can be written as follows

$$A = A_1 \cos a = w_0^+ a \cos a$$

where $a = \arctan(h_y / h_x)$ is the parameter characterising the dynamic roughness structure. If $A = 0$, then $a = p/2$. This case corresponds to the special type of the dynamic roughness composed of furrows elongated along the streamlines of the mean flow. If $a \neq p/2$, then $\tan a = h_y / h_x \approx l_x / l_y$. It can be considered as the relationship between the scales of the dynamic roughness elements in the X- and Y-directions. In this case a fluctuates around the mean value, $a = p/2$, and produces the velocity and pressure fluctuation. The velocity gradient on the wall fluctuates with a as follows

$$du^+ / dz^+ = w_0^+ a \cos a + A_2 \sin a$$

For $a = p/2$ we have $du^+ / dz^+ = A_2 = w_0 / w_0^*$. Therefore in this case the velocity gradient on the wall is given by

$$du^+ / dz^+ = (w_0 / w_0^*)(w_0^+ a \cos a + \sin a) \qquad (2.51)$$

The pressure gradient on the wall also depends on the second velocity scale w_0^+ as

$$p_x(0) = (2 r u_*^2 / l^+) a w_0^+ \qquad (2.52)$$

The velocity components near the smooth wall for $z \to 0$ can be written as follows (see Kutateladze [62])

$$u \cong K_1 z,\ \mathrm{v} \cong K_2 z,\ w \cong K_3 z^2,$$

where $K_i = K_i(x, y, t)$ are the viscous sublayer functions. Substituting the velocity approximation formulas in the x-component of momentum equation and supposing that $p = p(h)$ one can derive

$$z\frac{\partial K_1}{\partial t} - z\frac{p_x(0)}{r\,l^2}\cos a - zn\left(\frac{\partial^2 K_1}{\partial x^2}+\frac{\partial^2 K_1}{\partial y^2}\right) = z^2 o(1)$$

where $o(1)$ is the restricted value for $z=0$. Using equations (2.51)-(2.52) finally we have the dynamic roughness surface equation in case of the steady turbulent boundary layer over a smooth surface

$$\frac{\partial K}{\partial t} - \frac{2u_*^2 w_0^+ a}{n(l^+)^3}\cos a = n\left(\frac{\partial^2 K}{\partial x^2}+\frac{\partial^2 K}{\partial y^2}\right) \qquad (2.53)$$

where $K=(w_0/w_0^*)(w_0^+ a\cos a + \sin a)$.

The steady turbulent flow dynamic roughness is realised for $w_0 = w_0^*$. In this case equation (2.53) can be transformed into the quasi-linear differential equation

$$nK_a\left(\frac{\partial^2 a}{\partial x^2}+\frac{\partial^2 a}{\partial y^2}\right) + nK_{aa}\left(a_x^2 + a_y^2\right) = -\frac{2u_*^2 w_0^+ a}{n\,l^{+3}}\cos a + K_a\frac{\partial a}{\partial t} \qquad (2.54)$$

where $K_a = -w_0^+ a\sin a + \cos a$, $\quad K_{aa} = -w_0^+ a\cos a - \sin a$.

The point in which $K_a = 0$ is the singular point of the equation (2.54). In this point $\tan a_* = -(w_0^+ a)^{-1}$. As it has been estimated in the numerical experiments $w_0^+ a \approx -0.127$ for the mean flow, therefore $a_* \approx p/2 - w_0^+|a|$. For the stationary case, i.e. for $a \approx p/2$, the equation (2.54) can be written in the quasi-elliptical form

$$(1-\tilde{a})\left(\frac{\partial^2 \tilde{a}}{\partial x^2}+\frac{\partial^2 \tilde{a}}{\partial y^2}\right) - \left(\tilde{a}_x^2 + \tilde{a}_y^2\right) = -k_0^2 \tilde{a}\ , \qquad (2.55)$$

were $\tilde{a} = (a - p/2)/\left|w_0^+ a\right|$, $k_0^2 = 2u_*^2/n^2 l^{+3}$.

Using the function $\tilde{a} = \tilde{a}(x,y)$ one can calculate the dynamic roughness surface parameters as follows

$$h_x + \left|w_0^+ a\right|\tilde{a}h_y = 0, \quad h_t = w_0^*\sqrt{h_x^2 + h_y^2} \approx w_0^* h_y \qquad (2.56)$$

In the special case when $\tilde{a} << 1$, equation (2.55) has the periodical solution

$$\tilde{a} = a_0\cos(k_y y)$$

where a_0 is the amplitude, $k_y = \sqrt{2}(l^+)^{-3/2}u_*/n$ is the wave number in the y-direction. Therefore the transversal length scale of coherent structures can be estimated as

$$l_y^* = 2p/k_y = \sqrt{2}p\,l_0^{+3/2} n/u_* \approx 114n/u_*$$

The predicted length scale is in a good agreement with the experimental value, $l_y \approx 100 n / u_*$, obtained by Kline *et al.* [70]. This type of coherent structures corresponds to the furrows considered above.

In the special case when $|1 - \tilde{a}| << 1$, the periodical solution of the equation (2.55) is given by

$$\tilde{a} = 1 + a_0 \cos(k_y y)$$

where $k_y = \sqrt{2}(l^+)^{-3/2} u_* / a_0 n$. Therefore in this case l_y depends on the amplitude a_0. The periodical solution of the first equation (2.56) can be written as

$$h(x, y, t) = h(k_x x - k_y y_t - b(k_y y_t)), \qquad (2.57)$$

$$b(s) = \sum_{n=1}^{\infty} a_0^n \int \cos^n s ds$$

$$k_x = |w_0^+ a| k_y, \quad y_t = y + w_0^* t .$$

The transversal phase velocity of the dynamic roughness surface disturbances can be determined as $c_y = w_0^*$. The dynamic roughness length scale l_x depends on the amplitude a_0 as follows

$$l_x = l_y^* a_0 / |w_0^+ a| \approx 898 a_0 n / u_* .$$

For $a_0 \approx 1$ the estimated streamwise length scale of coherent structures agrees with the experimental value $l_x \approx 1000 n / u_*$ obtained by Blackwelder & Eckelmann [74], and discussed by Cantwell [52].

In this paper the problems of non-linear theory of turbulent boundary layer have been studied. The algorithm of numerical solution of the problem has been considered. Equation is deduced, connecting constants of non-linear theory.

A fundamental parameter of the turbulent boundary layer length has been determined, which coincides with the position of velocity peak of turbulence energy generation according to Klebanoff [49] and Laufer's [50] data. It is shown that the profile of an average velocity in the boundary layer can be described satisfactory, using only one constant. The Karman constant can be used for this purpose. The second constant of the logarithmic profile can be estimated within this theory. Velocity profile, calculated according to the model suggested, conforms well to the data of direct numerical modelling, to experimental data and models of other authors. Results of velocity intensity pulsation modelling have been presented, as well as their compliance with the results of direct numerical modelling and experimental data. A model of dynamic roughness in turbulent boundary layer has been suggested. It has also been shown that in a stationary case there are two types of periodic solutions. One solution corresponds to dynamic roughness in a kind of furrows, stretched along the main flow. The viscous flow over the structures is a physical mechanism of formation of logarithmic profile of velocity. The second solution corresponds to the perturbations of limited amplitude which have a limited length in the direction of the mean flow. It is shown that the parameters of dynamic roughness, having been calculated on the base of this model, coincide with the data of experiments.

(*To be continued*)

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